A TRANSIENT RESPONSE ANALYSIS OF A YAW STABILIZATION SYSTEM FOR THE HILLER YROE-1 ROTORCYCLE

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ABSTRACT

The unusual flight characteristics of the Hiller YROE-1 Rotorcycle dictates the installation of a yaw stabilization system to relieve the pilot of flight control problems in yaw and to increase his effectiveness in the performance of his assigned flight mission. This paper develops system component transfer functions and then presents a transient response analysis of a yaw stabilization system proposed by Royal Industries Incorporated for installation on the Rotorcycle. Several modifications of the original system are proposed by the authors in order to improve performance to meet Hiller specifications and analysis of these modifications is also presented.

The writers wish to express their appreciation for the courtesies and cooperation extended to them by the personnel they had the pleasure of working with at Hiller Aircraft Corporation and Royal Industries Incorporated.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. Introduction.

The Hiller YROE-1 Rotorcycle shown in Fig. 1 is a foldable transportable aircraft, with full helicopter capabilities, being built by the Hiller Aircraft Corporation of Palo Alto, California, for the U. S. Marine Corps. The design concept behind the Rotorcycle is to provide the Marine Corps with a small helicopter which can be used by units in the field for spotting, observation, courier work, etc., but which can be easily transported, maintained, and flown under field combat conditions. The experimental machine already constructed can be assembled and flown by one man in less than ten minutes.

The flight characteristics of the Rotorcycle are to be such that it can be flown by personnel whose flight training has been limited specifically to the Rotorcycle and who have had no previous training as aircraft pilots. This is a severe requirement since the conventional helicopter is inherently unstable about all three flight axes, (longitudinal, lateral and directional), and must be continuously "flown" by the pilot through controls that demand constant use of both hands and both feet. To make the Rotorcycle stable and simple to fly, two flight control problems normally handled by the skilled helicopter pilot must be eliminated from the control problems that the relatively unskilled Rotorcycle pilot will face. These two flight control problems are collective pitch-throttle coordination and engine torque compensation.

The most difficult problem for the novice is the coordination of throttle application with main rotor collective pitch to maintain constant

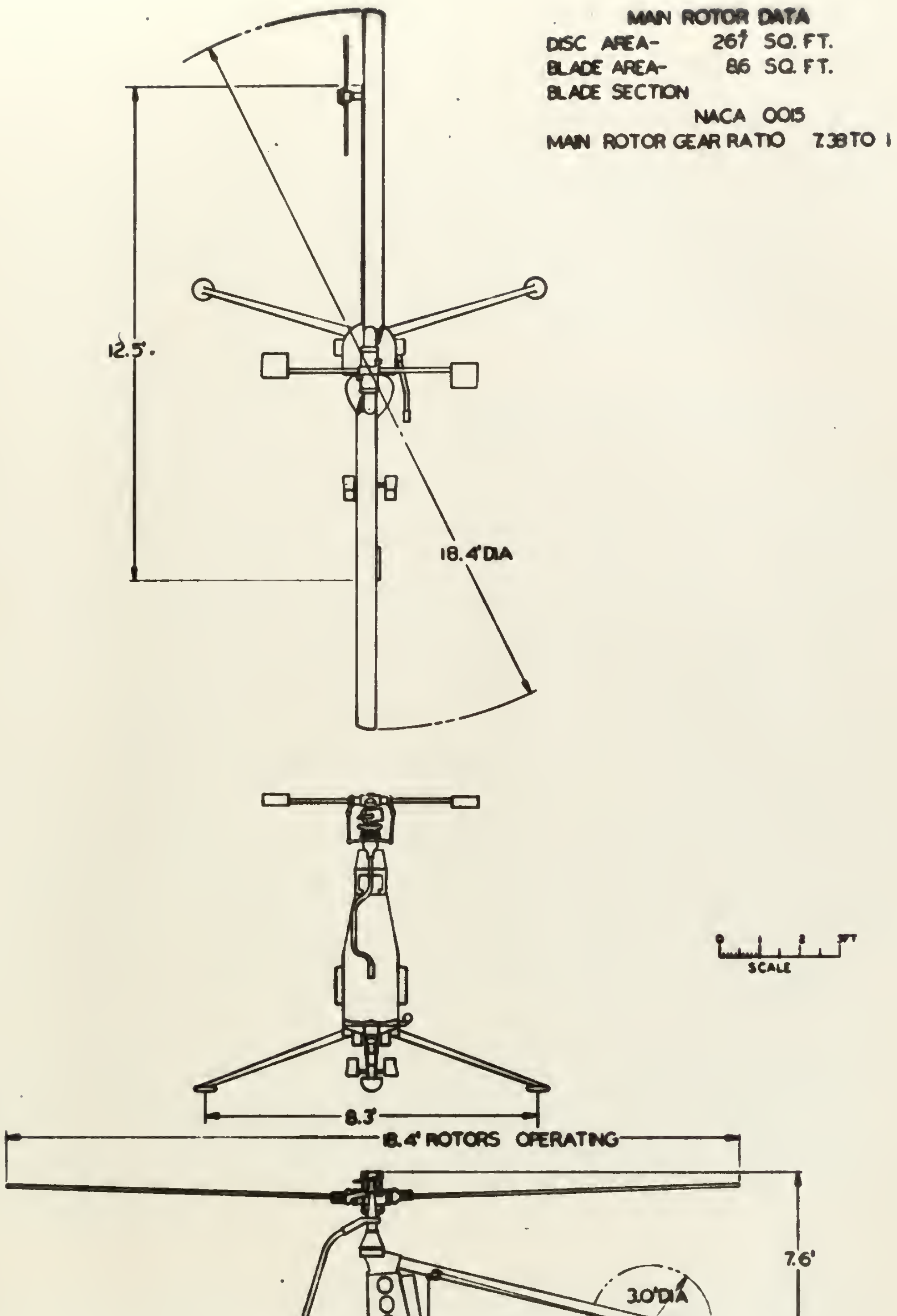


Fig. 1 DESCRIPTIVE ARRANGEMENT

main rotor RPM.[1] Main rotor RPM must be kept within a certain critical operating range during flight in order that the rotor blades do not stall and thus lose lift, or fail structurally due to excessive air and/or centrifugal loads. On the Rotorcycle a throttle governor will handle this collective pitch-throttle problem by means of two settings; 1) governing to maintain established RPM, and 2) idle condition for starting and practice autorotation.[2] Thus the Rotorcycle pilot will have just two throttle settings, idle and full open.

The second control problem is that of engine torque compensation. The Rotorcycle is classified as a single rotor helicopter. In single-rotor helicopters the reaction torque of the main rotor upon the aircraft fuselage is counterbalanced by the moment about the main rotor axis produced by the thrust of a tail rotor. This is mounted on a boom aft of the main rotor. Directional control is maintained by the movement of rudder pedals by the pilot which control the tail rotor blade pitch and thus the tail rotor thrust. As the application of engine torque varies there is a change of thrust required from the tail rotor if constant aircraft heading is to be maintained. This is particularly important in the event of actual or simulated autorotation which involves an immediate

[1]Collective pitch refers to angle of attack or pitch of the main rotor blades. A change in collective pitch changes the blade pitch simultaneously, i.e. collectively. To maintain main rotor RPM within the critical range requires appropriate throttle variation with collective pitch changes, i.e. collective pitch increase requires throttle increase and vice versa.

[2]Autorotation refers to the rotation of main rotor blades solely through lift on the blades. When the main rotor is not being driven by the helicopter engine, as in the case of an engine failure, with the blades at full low pitch the rotor will rotate freely. The resulting lift will allow the helicopter pilot to make a rapid but controlled descent to the ground. Upon engine failure the pilot must make an immediate transition to autorotation to maintain controlled flight.

cessation of engine torque. Unless rudder control is immediately applied, the tail rotor will spin or yaw the aircraft about its vertical axis in the direction of main rotor rotation. This is particularly true of the Rotorcycle which has a low moment of inertia about its vertical axis and is very yaw sensitive. Consequently Hiller Aircraft proposed that a yaw stabilization system be installed on the Rotorcycle. This will be an electronic stabilization system that will control aircraft yaw by varying the pitch of the tail rotor blades. It will be sensitive enough to maintain Rotorcycle yaw angles and yaw rates within specified limits[3] under the most adverse flight conditions. These limits have been defined so that the Rotorcycle pilot is essentially relieved of any of the control problems associated with engine torque compensation.

The yaw stabilization system presently planned for the Rotorcycle was proposed to Hiller Aircraft by Royal Industries Incorporated of Alhambra, California. This paper presents a transient response analysis of the system as proposed by Royal Industries, using representative component values, and then investigates several modifications proposed by the authors to further improve system performance.

Work on this thesis was accomplished during nine weeks of the summer period between the second and third years of the Ordnance Engineering (Guided Missiles) Curriculum ending in December of 1959. The authors spent six weeks at the Hiller Aircraft plant at Palo Alto, two weeks at the Naval Postgraduate School at Monterey, and four days at the Royal

[3] Hiller Aircraft Engineering Report No. 59-23, Proposal to the Bureau of Aeronautics for Evaluation of Yaw Stabilizer and Throttle Governor for the XROE-1 Rotorcycle.

Industries plant at Alhambra. This time was spent in familiarization with the problem, study of the helicopter and various system components, and, finally, system analysis.

2. Single-Rotor Helicopter and Rotorcycle Directional Stability.

The tail rotor of a conventionally powered single-rotor helicopter has two purposes—to counteract the main rotor torque and fuselage yawing moments and to maneuver the helicopter directionally. Helicopter flying quality studies have indicated a minimum desirable response of 3 degrees yaw in the first second following a 1-inch displacement of the pedals while hovering in zero wind. In addition to indicating a minimum desirable response value, these studies have also indicated the existence of a maximum desirable response value. When large pedal friction and out-of-trim forces are present, the maximum desirable response value is indicated to be approximately 10 degrees of yaw in the first second following a 1-inch step displacement of the pedals while hovering in zero wind. When pedal friction and out-of-trim forces are relatively small, a maximum desirable value of 2 to 4 times as large as the 10 degree value is indicated.[4]

Although the Rotorcycle falls in the category of a conventionally powered single-rotor helicopter, the requirement that it be foldable and transportable results in poorer directional stability characteristics than are found in the larger helicopters in more general use. The poor directional stability characteristics of the Rotorcycle result from a very low moment of inertia about the vertical axis through the center of gravity and the short tail boom, i.e. moment arm, available for the thrust vector of the tail rotor.

A change in main rotor torque, unless corrected for by a counter-act-

[4] Amer, K. B. and Gessow, A., Charts for Estimating Tail Rotor Contribution to Helicopter Directional Stability and Control in Low-Speed Flight, NACA Report 1216, 1955.

ing moment of the tail rotor, will cause a single-rotor helicopter to rotate or yaw at some rate about its vertical axis through the center of gravity. Initially the rate of yaw is dependent upon the helicopter inertia about its vertical axis whereas later it is primarily dependent upon the damping of the tail rotor. Engine power variations produce changes in main rotor torque and the most severe change in rotor torque results from complete engine failure. An interesting comparison can be made between computed yaw angles at 1 second after complete engine failure during hovering for the Hiller H-23E, a single-rotor helicopter produced for commercial use, and the Rotorcycle if no corrective action is taken through use of the tail rotor. At the end of 1 second the H-23E will have yawed approximately 35 degrees, the Rotorcycle 255 degrees.

Because of the short tail boom and correspondingly short tail rotor moment arm on the Rotorcycle, to attain the necessary counteracting torque the tail rotor is designed to be very powerful. Small changes in the tail rotor pitch produce large thrust changes. Consequently the Rotorcycle is very sensitive directionally to tail rotor blade pitch changes, much more so than the Hiller H-23E helicopter. A 1-inch right pedal displacement will cause the Rotorcycle to yaw to the right approximately 245 degrees at the end of the first second. But in spite of this control sensitivity, if main rotor torque should vary in any way, the tail rotor blade pitch must be changed to counteract the change in main rotor torque almost instantaneously if the Rotorcycle is to maintain a constant heading or not yaw excessively. In the case of a sudden power application, i.e. a ramp increase of main rotor torque, within certain limits the pilot can be expected to counteract the torque change through pedal movement since

he himself initiated the change. In the case of an unexpected and complete engine failure, i.e. a maximum step decrease of main rotor torque, the pilot's reaction time to counteract the torque change is the important factor. The project pilot for the experimental Rotorcycle found that the helicopter yawed approximately 90 degrees when he "led" with rudder pedal correction during simulated engine failure flight tests. An infinitely large amount of control sensitivity will be of little value if it is not applied soon enough. Since the Rotorcycle will yaw or rotate at such an excessively high rate with complete engine failure, a pilot could become completely disoriented and be unable to make the transition to autorotation for an emergency landing.

3. Yaw Stabilization of the YROE-1

From the discussion presented in section 2, it can be seen that if the Rotorcycle is to be a simple machine capable of being flown by personnel with limited flight training, some means of eliminating the directional control problems must be devised. The logical solution would be some device that would sense any change in main rotor torque and activate a servo system to drive the tail rotor blades to a pitch that would produce a counteracting tail rotor torque. The application of this corrective torque would be such that low yaw rates and small yaw angles would result that could be easily controlled or even neglected by the pilot.

Hiller Aircraft accordingly asked for proposals from various interested contractors for a yaw stability system for the Rotorcycle to meet the following requirements[5]:

1. With a rate of change of torque equivalent to an increase in engine power from zero (0) to one hundred percent (100%) in 2 seconds at zero (0) forward speed, the maximum deviation from course shall be ± 5 degrees. Maximum yaw rate shall be 0.2 radians per second.
2. With a rate of change of torque equivalent to an increase in engine power from zero (0) to one hundred percent (100%) in 0.2 seconds at zero (0) forward speed the maximum deviation from course shall be ± 15 degrees. Maximum yaw rate shall be 0.4 radians per second.
3. With instantaneous power failure in hovering, the maximum

[5]Hiller Aircraft Engineering Report No. 59-23, op.cit., p. 6

deviation from course shall be 30 degrees. Maximum yaw rate shall be 0.6 radians per second.

Of the several proposals received, only two received serious consideration by Hiller. One was submitted by the Astronics Division of Lear, Incorporated, Santa Monica, California and the other by Royal Industries Incorporated, Alhambra, California. Both systems were to use rate gyros to sense yaw rate and Lear-designed and built magnetic clutches to drive the tail rotor blades to the required pitch angles. The Royal system was to be a series servo system while the Lear system was to be a parallel servo system. The prime advantage of a series servo is that any movement of the servo is not reflected at the pilot's pedals. The prime disadvantage is that in the event of failure of the series servo, it must be centered and locked. On the other hand, the prime advantage of a parallel servo is that in the event of failure, it need only be declutched, or put into free wheeling. The disadvantage is that all servo movements are reflected at the pilot's pedals. Thus any input signal to the servo for stability augmentation will produce a constant jittering of the pedals that could be disconcerting to the pilot.[6] Figs. 2 and 3 illustrate simple series and parallel servos.

Hiller Aircraft selected the system proposed by Royal Industries. It had the advantages of a series system, met the disadvantage of a series system with what appeared to be a good fail-safe mechanism in the event of system power failure, and allowed the pilot to insert change of heading commands by means of pedal movement through the stabilization system. This meant that the system would work full time unless turned off by the

[6] Strobel, L. W., What's Ahead for Piloted Flight Control Systems, SAE Journal, Vol. 66, No. 11, Nov. 1958, p. 93.

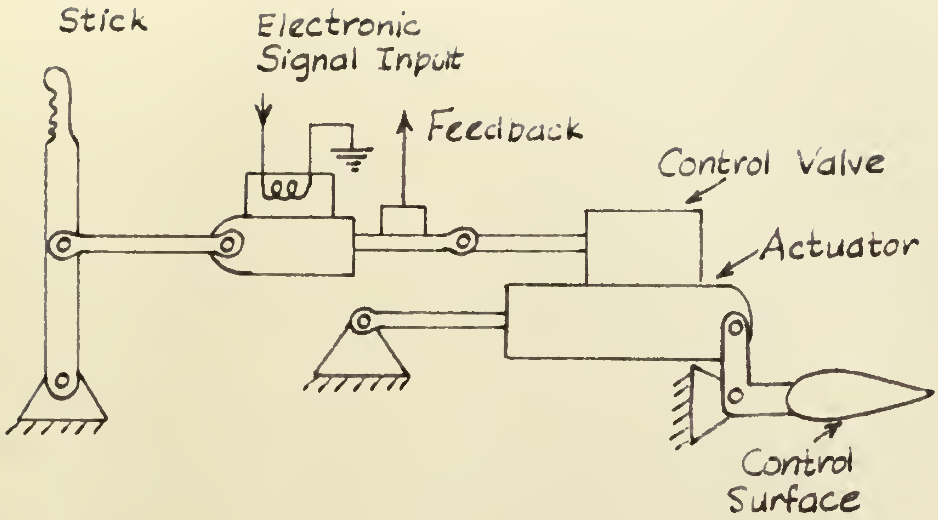


Fig. 2. Simple Series Servo.

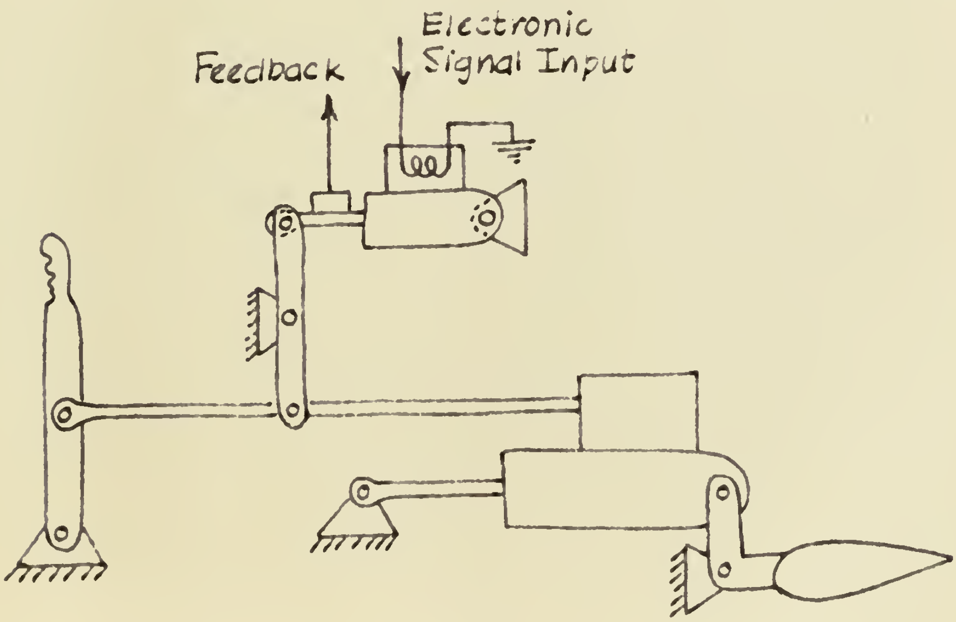
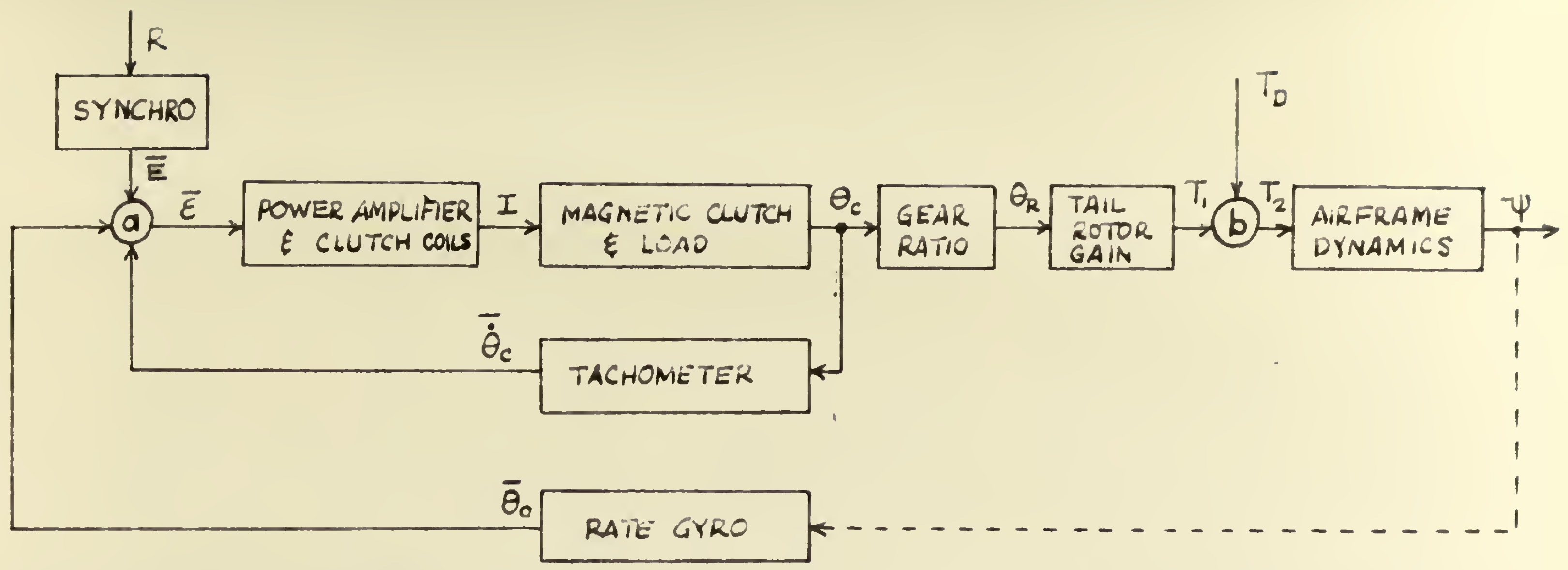


Fig. 3. Simple Parallel Servo.

pilot or made inoperative because of a system power failure.

Basically the system will operate as follows. A rate gyro, which has its input axis coincident with the helicopter yaw axis, will sense helicopter yaw rate. In the event of a torque disturbance, the yaw rate signal will be fed back to a servo amplifier. The signal from the amplifier will energize the appropriate magnetic clutch which will change the tail rotor pitch angle in the proper direction to counteract the torque acting upon the helicopter to reduce the yaw rate to some acceptable value. Reduction of the yaw rate will in turn reduce the final angle of yaw for a given torque disturbance.

A block diagram showing the arrangement of the system physical components is presented in Fig. 4. Before any system analysis could be attempted, the transfer functions of these components had to be determined; this is done in the section following.



R = Rudder pedal displacement (inches).

$\bar{E}$ = Command input from rudder pedal synchro (volts).

$\bar{\mathcal{E}}$ = Error signal (volts).

I = Current to magnetic clutches (ma).

θ_c = Magnetic clutch output shaft angle (radians).

θ_R = Tail rotor blade pitch angle (radians).

T_1 = Torque exerted on airframe about vertical axis by tail rotor (ft-lbs).

T_D = Disturbance torque on airframe about vertical axis from torque variation or wind gust (ft-lbs).

T_2 = Resultant torque on airframe about vertical axis due to tail rotor and disturbance (ft-lbs).

ψ = Helicopter angle of yaw (radians).

$\bar{\dot{\theta}}_c$ = Tachometer output (volts).

$\bar{\theta}_o$ = Rate gyro output axis angular displacement (volts).

Fig. 4. Descriptive Block Diagram.

4. Component Transfer Functions.

Airframe Dynamics

By assuming a one-degree-of-freedom system, the equation of motion of a helicopter in yaw is[7]

$$T = J_h \frac{d^2\psi}{dt^2} + f_h \frac{d\psi}{dt} + K_h \psi$$

where T is the torque applied to the airframe, J_h is the airframe inertia about the vertical axis, f_h is the tail rotor damping, K_h is the tail rotor directional stability (due to weathervaning), and ψ is helicopter yaw angle.

Taking the Laplace transform of the above equation and putting it into transfer function form yields

$$\frac{\psi(s)}{T(s)} = \frac{1}{J_h s^2 + f_h s + K_h}$$

The following values apply to the Rotorcycle for the two conditions of interest.[8]

Flight Condition	J_h	f_h	K_h
Hovering	25 slug ft.^{2}	32.8 $\frac{\text{ft-lb}}{\text{rad/sec}}$	0
Forward Flight	25 slug ft.^{2}	46.7 $\frac{\text{ft-lb}}{\text{rad/sec}}$	688 $\frac{\text{ft-lb}}{\text{rad}}$

When these values are substituted into the transfer function equation the following expressions relating applied torque and helicopter yaw are obtained

[7]Amer, op. cit., p. 20.

[8]Hiller Aircraft Engineering Report No. 59-23, op. cit.

Hovering: $$\frac{\psi(s)}{T(s)} = \frac{1}{25s^2 + 32.8s} = \frac{.04}{s(s+1.31)}$$

Forward Flight: (70 MPH) $$\frac{\psi(s)}{T(s)} = \frac{1}{25s^2 + 46.7s + 688}$$

$$= \frac{.04}{s^2 + 1.87s + 27.5} = \frac{.04}{(s+0.94+j5.2)(s+0.94-j5.2)}$$

Rate Gyro

A rate gyro is a single-degree-of-freedom gyro with an elastic restraint on its movement about the output axis so that the angular deflection of the output axis is proportional to the angular velocity about the input axis. The output of the rate gyro as installed on the Rotorcycle is a voltage proportional to the deflection of the output axis or to the angular velocity of the helicopter about its vertical axis.

If the suspension is sufficiently stiff mechanically for input motion, the response of the gyro approaches that given by only the dynamics of the output axis and the transfer function for the rate gyro becomes[9]

$$\frac{\bar{\theta}_o(s)}{\psi(s)} = \frac{\dfrac{K_g H s}{K_o}}{\dfrac{J_o}{K_o}s^2 + \dfrac{f_o}{K_o}s + 1}$$

[9]Truxal, J. G., Control Engineer's Handbook, McGraw-Hill Book Co., 1958, Chap. 17, pp. 45-48.

where

$\bar{\theta}_o$ = a voltage proportional to the angular deflection of the gyro output axis

ψ = helicopter yaw angle

K_g = gyro sensitivity

H = gyro angular momentum about the spin axis

K_o = elastic constant of output axis

J_o = moment of inertia about output axis

f_o = viscous damping of output axis

The foregoing equation can also be expressed as

$$\frac{\bar{\theta}_o(s)}{\psi(s)} = \frac{\dfrac{K_g H s}{J_o}}{s^2 + 2\zeta_o \omega_o + \omega_o^2}$$

where $\zeta_o = \dfrac{f_o}{2\sqrt{J_o K_o}}$ = damping ratio about output axis and

$$\omega_o = \sqrt{\frac{K_o}{J_o}}$$ = natural frequency about output axis.

Taking typical values of H, J_o, ζ_o, and ω_o for a Lear series 2157 rate gyro[10] and assuming a value for K_g of 3 volts per degree the transfer function becomes

$$\frac{\bar{\theta}_o(s)}{\psi(s)} = \frac{246{,}000\ s}{s^2 + 703 s + 252{,}900} = \frac{246{,}000\ s}{(s+353+j359)(s+353-j359)}$$

[10]Lear Product Data Sheet 116-5.

Magnetic Clutch and Load

A magnetic powder clutch transmits torque through the shear resistance of iron powder which fills the gap between the faces of the clutch. This powder "solidifies" when the small particles of powder cling together in the presence of magnetic flux. Torque may be controlled smoothly by changing the magnetic field strength to vary the consistency of the mixture and therefore its mechanical resistance.

A single clutch can exert torque only in the direction of its drive motor but if the motor drives two clutches in opposite directions the net torque on the output shaft is equal to the difference of the opposing torques of the two individual clutches. A double-clutch unit can drive in either direction depending on which clutch is energized. The clutch coils are the balanced load for the push-pull output stage of an electronic amplifier. This push-pull arrangement of the coils and of the clutches improves the linearity of the torque-current characteristic and when the bias-current in each coil is properly adjusted, the output torque is proportional to the differential current in the clutch coil circuit. The improved linearity is obtained by sacrificing some energy, because the quiescent coil current results in opposing clutch torques at zero signal (differential coil current).

An experimental determination[11] of the dynamic response of a magnetic-fluid clutch indicates the following transfer function

$$\frac{T(s)}{I(s)} = \frac{K_c}{T_c s + 1}$$

where T(s) is the Laplace transform of the clutch output torque, I(s) is the transform of the coil current, s is the Laplace complex variable,

[11] Parziale, A. J. and Tilton, P. D., Characteristics of Some Magnetic Fluid Clutch Servomechanisms, AIEE Proceedings, Vol. 69, 1950.

K_c is the steady-state torque-current sensitivity, and T_c is a characteristic time constant of the clutch response.

The load dynamics of the double-clutch servomotor are taken into account by using the foregoing equation to equate the applied torque to the inertia and damping load torques, which gives[12]

$$\frac{\theta_c(s)}{I(s)} = \frac{\frac{K_c}{f}}{s(T_c s + 1)(\frac{J}{f} s + 1)}$$

where J is the load inertia, f is the coefficient of equivalent damping, and $\theta_c(s)$ is the transform of the output shaft position.

Determination of the tail rotor load characteristics proved to be very difficult because no method was available to experimentally determine these values. The load consists of the tail rotor and a conventional cable system. The exact value of the load inertia (J) is unknown. The actual value of the moment of inertia of a single rotor blade about the longitudinal axis is 0.805 lb-in^2. It was the considered opinion of Hiller engineers that the load inertia is small enough to be neglected in comparison with the coefficient of equivalent damping (f). If a torque is applied to the tail rotor and the rotor is characterized only by a coefficient of equivalent damping the torque equilibrium equation is

$$T = f \frac{d\theta_R}{dt}$$

where $\frac{d\theta_R}{dt}$ is the time derivative of tail rotor pitch position. Taking the Laplace transform of the above equation yields

$$T(s) = f s \theta_R(s)$$

$$\therefore \quad \theta_R(s) = \frac{T(s)}{f s}$$

Hiller specifications state that maximum estimated net collective pitch

[12] Ibid.

control torque load is 77 inch-pounds.[13] If this torque is applied to the foregoing equation as a step forcing function

$$\theta_R(s) = \frac{77}{f s^2}$$

taking the inverse transform

$$\theta_R(t) = \frac{77\,t}{f}$$

and noting that

$$\frac{d\theta_R}{dt} = \frac{\theta_R(t)}{t} = \frac{77}{f}$$

the coefficient of equivalent damping can be expressed as

$$f = \frac{77}{\frac{d\theta_R}{dt}}$$

It now becomes necessary to make an estimate for the value of

$$\frac{d\theta_R}{dt}$$

when the tail rotor is subjected to a torque step forcing function of 77 inch-pounds. The tail rotor pitch range is 23 degrees and it is believed that a reasonable approximation for the time required for the tail rotor to travel through this range would be about 0.5 seconds. Therefore, if the value of $\frac{d\theta_R}{dt}$ is approximated as one radian per second, the value of the coefficient of equivalent damping becomes

$$f = 77 \ \frac{\text{in.-lb.}}{\text{rad./sec.}}$$

The gear ratio (N) between the magnetic clutch and the tail rotor is 0.6. The damping coefficient as reflected to the clutch shaft is

$$N^2 f = (0.6)^2(77) = 27.8 \ \frac{\text{in.-lb.}}{\text{rad/sec.}}$$

[13]Hiller Aircraft Engineering Report No. 59-23, op. cit.

In the following analysis the coefficient of equivalent damping at the clutch will be approximated as

$$30 \ \frac{\text{in.-lb.}}{\text{rad./sec.}}$$

The value of the characteristic time constant of the clutch response (T_c) is on the order of several milliseconds[14] and may be considered negligible in comparison with the time constant of the helicopter airframe.

The value of the double-clutch torque-current sensitivity (K_c) was obtained from a typical torque-current plot for biased operation of a pair of clutches showing net torque as linearly proportional to differential clutch coil current.[15] This value was 5.72 $\frac{\text{in.-lb.}}{\text{ma.}}$

Under the foregoing assumptions and conditions the transfer function for the double clutch and load becomes

$$\frac{\theta_c(s)}{I(s)} = \frac{K_c/f}{s(T_c s+1)(J/f \, s+1)} = \frac{5.72/30}{s} = \frac{0.191}{s}$$

<u>Amplifier</u>

The amplifier input is the error voltage and the output is the clutch coil current. An amplifier and clutch coil circuit time constant is present but is very small and will be neglected. The transfer function for the amplifier will be taken as a pure gain (K_a) expressed in milliamperes per volt.

<u>Tail Rotor Gain</u>

The torque produced on the helicopter airframe by the tail rotor is dependent on the pitch angle of the tail rotor blades (the tail rotor

[14] Graham, D., Magnetic Clutches Add Muscle to Electronic Servos, Space Aeronautics, April, 1959.

[15] Ibid.

rotates at a constant RPM). The value of the tail rotor control power or "tail rotor gain" is 1792 foot-pounds per radian of tail rotor pitch when the helicopter is in the hovering condition and is 2550 foot-pounds per radian of tail rotor pitch when the helicopter is in forward flight at a velocity of 70 miles per hour.[16]

Pedal-Airframe Differential Position Pick-off

An input command signal is generated when the pilot depresses a rudder pedal. A synchro attached to the pedal and the airframe senses this displacement and generates a voltage proportional to the displacement which is added to the feedback voltage to produce the error voltage. The value of this synchro gain (K_s) is adjustable within certain limits. In the following analysis the command input to the system will be taken as the voltage output of this synchro because changing the value of K_s serves only to adjust the sensitivity of the rudder pedals (i.e. the yaw rate produced by a given pedal deflection) and does not otherwise affect the functioning of the servo system.

[16] Hiller Aircraft Engineering Report No. 59-23, op.cit.

5. System Analysis

The two helicopter conditions under consideration are (1) hovering flight and (2) forward flight at a velocity of 70 miles per hour. These two conditions result in different transfer functions for the airframe as shown in Section 4. The primary difference between the two conditions lies in the fact that no tail rotor directional stability, or "weathervaning" effect (spring constant), is present during hovering. It thus becomes obvious that the hovering condition must be the more critical condition when attempting to prevent yaw caused by a torque disturbance. One can reason that any yaw displacement experienced in the forward flight condition would be less severe were the same torque disturbance applied while in the hovering condition. Accordingly, the major portion of the system analysis is concerned with the Rotorcycle in the hovering condition.

The system is subjected to two kinds of inputs which are applied at different nodes of the system. A yaw command signal is originated when the pilot depresses a rudder pedal and this signal enters the main servo path at node a as shown in Fig. 4. The other input to the system consists of a torque disturbance produced by a rapid change in main rotor power as previously described. This disturbance torque is applied at node b and is added algebraically to the torque being produced by the tail rotor, the resultant net torque is that actually applied to the airframe. The primary purpose of the servo system is to control the yaw produced by a torque disturbance, therefore the response to this type of input should be the major criterion in the design of the system. The response to a command input is also very important, of course, but will be considered only after system response to a disturbance input is obtained.

The block diagram of Fig. 5 was obtained by combining the component transfer functions for the hovering condition set forth in the preceding section with the descriptive block diagram of Fig. 4. This diagram reduces to that shown in Fig. 6 which is in standard form for application of the determinental method of analysis.[17]

The characteristic determinant of the system is

$$\Delta = \begin{vmatrix} 1+\left(\frac{204\,K_a}{s}\right)\left(\frac{K_t\,s}{1075}\right) & \frac{.04}{s(s+1.31)}\left(\frac{246\,000\,s}{s^2+703\,s+252900}\right) \\ -\frac{204\,K_a}{s} & 1 \end{vmatrix}$$

and the output at node b in response to a step torque disturbance input of 330 ft-lb (corresponding to power failure) is given by

$$b = \frac{\begin{vmatrix} 1+\left(\frac{204\,K_a}{s}\right)\left(\frac{K_t\,s}{1075}\right) & 0 \\ -\frac{204\,K_a}{s} & \frac{330}{s} \end{vmatrix}}{\Delta}$$

Multiplication by the output block (representing the helicopter airframe) gives the resulting yaw of the helicopter and application of the Final Value

[17] Chu, Y., A Generalized Theory of Linear Multi-Loop Automatic Control Systems, Doctoral Dissertation, MIT, 1953.

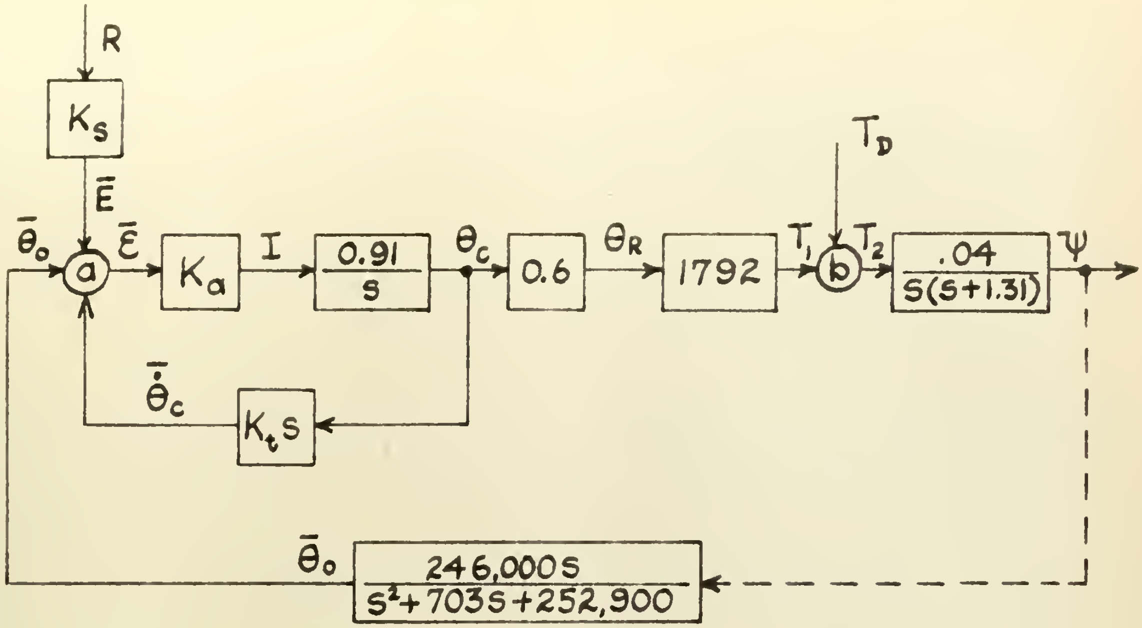


Fig. 5 Block diagram with transfer functions.

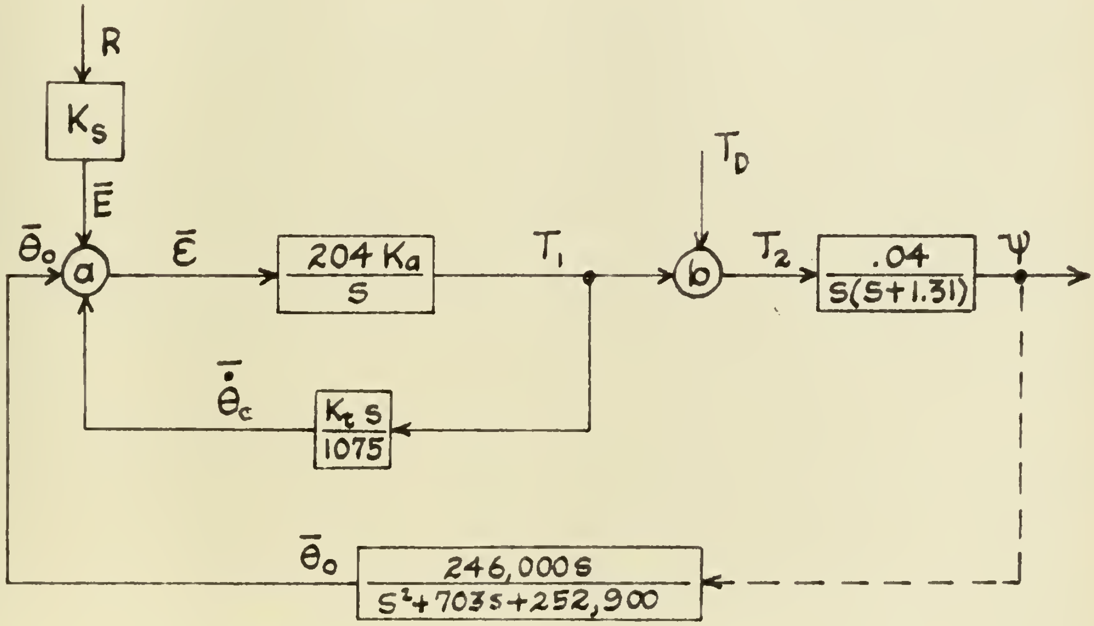


Fig. 6 Simplified block diagram with transfer functions.

Theorem (letting $s \longrightarrow 0$) gives the steady-state angle of yaw (as $t \longrightarrow \infty$). If the value of tachometer gain (K_t) is taken as 0.6 volts per radian per second as a convenient value, the steady-state yaw angle as a function of amplifier gain is

$$\left(\frac{13.2 + 1.45\,K_a}{7.95\,K_a}\right) 57.3 \quad \text{degrees}$$

and it is seen that as K_a is increased the steady-state output decreases. As $K_a \longrightarrow \infty$, the limiting minimum value of the steady-state yaw angle approaches

$$\left(\frac{1.45}{7.95}\right) 57.3 = 10.45 \text{ degrees}$$

If K_a is taken as 25 $\frac{\text{ma}}{\text{volt}}$ the steady-state yaw angle is

$$\left[\frac{13.2 + 1.45(25)}{7.95(25)}\right] 57.3 = 14.25 \text{ degrees}$$

Following the same procedure with $K_t = 0.1$ gives limiting values of 1.81 degrees as $K_a \longrightarrow \infty$ and 5.61 degrees when $K_a = 25$. It is noted that the steady-state yaw angle decreases as K_t is decreased. Letting $K_t = 0$ (corresponding to no tachometer feedback) produces limiting values of zero degrees as $K_a \longrightarrow \infty$ and 3.81 degrees when $K_a = 25$. The apparent conclusion is that no tachometer feedback is the best arrangement from the steady-state viewpoint.

The same conclusion can be reached in a qualitative manner through the following reasoning. The minor loop of Fig. 6 (the tachometer feedback loop) can be represented as shown in Fig. 7 where the output of this minor loop is the corrective torque (T_1) sent to node b and the input is the sum of rudder pedal synchro and rate gyro outputs.

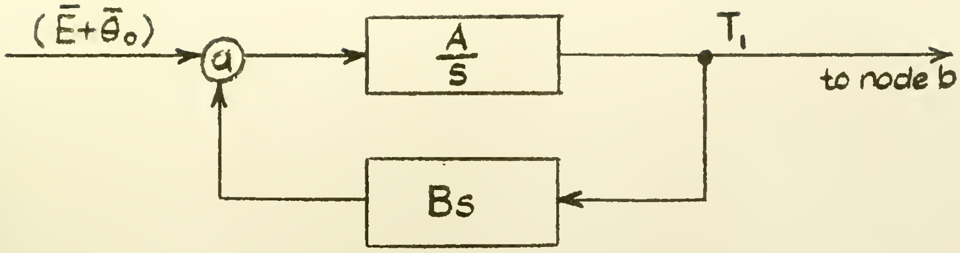


Fig. 7
Tachometer Loop

When the airframe is subjected to a disturbance torque and commences to yaw, it is desirable to have the most rapid increase in corrective torque that is obtainable in order to stop the yawing motion. The closed loop transfer function of the minor loop in Fig. 7 is

$$\frac{T_1(s)}{(\bar{E}+\bar{\theta}_o)(s)} = \frac{\frac{A}{s}}{1+\frac{A}{s}(\beta s)} = \left(\frac{A}{1+A\beta}\right)\frac{1}{s}$$

and therefore the minor loop can be represented by the equivalent block shown in Fig. 8.

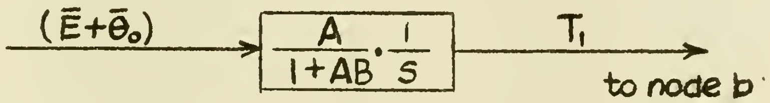


Fig. 8
Equivalent Tachometer Loop

The rate of increase of corrective torque for a given input can be obtained by differentiating (multiplying by s) both sides of the equation for the closed loop transfer function of the tachometer feedback loop.

$$\frac{s\,T_1(s)}{(\bar{E}+\bar{\theta}_o)(s)} = s\left(\frac{A}{1+AB}\right)\frac{1}{s} = \frac{A}{1+AB}$$

It is seen that this rate depends upon the value of $\frac{A}{1 + AB}$ which is obviously at a maximum value equal to A when B = 0 (corresponding to no tachometer feedback).

It may further be reasoned that, if all of the other system components are fixed in value, the variation in steady-state yaw angle is caused only by the variation in the equivalent block of the minor loop, or $\frac{A}{1 + AB} \cdot \frac{1}{s}$. Therefore, if one chooses a desired steady-state output, the value of $\frac{A}{1 + AB}$ is fixed and the minor loop must always reduce to the identical equivalent block regardless of the individual values of A and B (corresponding to the values of K_a and K_t). If the minor loop equivalent block is always the same for a given steady-state output of the system, the transient response of the system must always be the same for a given steady-state output.

Since the best steady-state response is obtained with no tachometer feedback and the transient response is unaffected by tachometer feedback for a given steady-state response, it may be concluded that tachometer feedback should be eliminated from the system.

If tachometer feedback is removed, the system block diagram becomes that shown in Fig. 9.

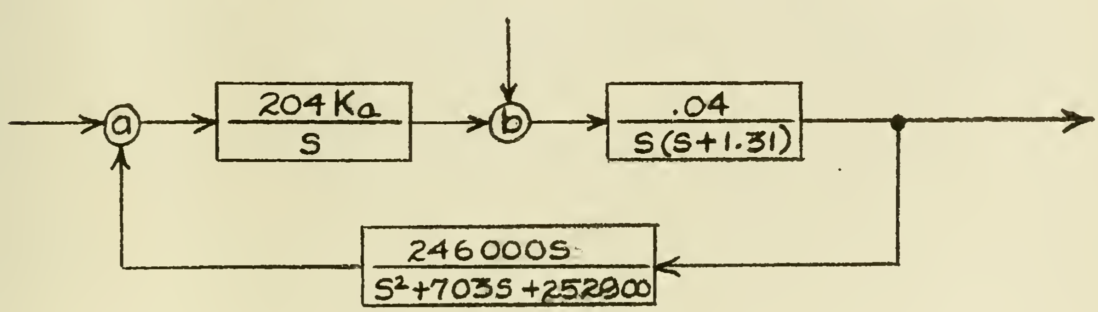


Fig. 9

System Block Diagram With Tachometer Removed

The open-loop transfer function is

$$\frac{(204\,K_a)(.04)(246\,000\,s)}{s^2(s+1.31)(s^2+703s+252900)} = \frac{2,010,000\,K_a\,s}{s^2(s+1.31)(s^2+703s+252900)}$$

and the closed-loop transfer function for an input at node b is

$$\frac{\dfrac{.04}{s(s+1.31)}}{1+\dfrac{2,010,000\,K_a\,s}{s^2(s+1.31)(s^2+703s+252900)}} = \frac{.04\,(s^2+703s+252900)}{s(s+1.31)(s^2+703s+252900)+2010000K_a}$$

If the Final Value Theorem is applied to this expression, the steady-state output in response to a step torque disturbance input of 330 ft-lbs is given in terms of amplifier gain by the following expression

$$\lim_{s\to 0} s\left[\frac{.04\,(s^2+703s+252900)}{s(s+1.31)(s^2+703s+252900)+2,010,000\,K_a}\right]\left(\frac{330}{s}\right)$$

$$= \frac{.04(252900)(330)}{2,010,000\,K_a} = \frac{1.66}{K_a}.$$

For a steady-state output of 30 degrees, the amplifier gain must be

$$K_a = \frac{1.66\,(57.3)}{30} = 3.18\ \frac{\text{ma.}}{\text{volt}}$$

Corresponding values of K_a for steady-state values of 15 degrees and 5 degrees are 6.36 and 19.1 respectively.

The root locus plot for this system is shown in Fig. 10 with several gain points indicated on the locus. The open-loop complex poles resulting from the rate gyro are far enough to the left to have negligible effect on the locus in the region of interest near the origin and therefore this portion of the locus is, for all practical purposes, a straight vertical line intersecting the real axis at -0.65. It is seen that this system has two complex pairs of roots (closed-loop poles) and one complex pair of closed-loop zeros. One pair of poles and one pair of zeros are located so far to the left that their effect on the system transient response is negligible; this leaves only one pair of complex closed-loop poles and no closed-loop zeros which is a second-order system.

It is apparent from an inspection of the root locus that, for a value of K_a required to give satisfactory steady-state response, the root location will be such as to produce a highly oscillatory transient response. A value of K_a of 6.22 $\frac{\text{ma}}{\text{volt}}$ produces a steady-state output of 15.3 degrees and locates the roots at $-0.65 \pm j7$. The transform of the yaw angle response to a 330 ft-lb step torque disturbance is

$$\psi(s) = \frac{.04}{(s+0.65+j7)(s+0.65-j7)}\left(\frac{330}{s}\right)$$

$$= \frac{13.2}{s\left[(s+0.65)^2+7^2\right]}$$

Taking the inverse transform,[18] the output is given by

$$\psi(t) = 0.2675 + 0.2685\, e^{-0.65t} \sin\left(7t - 95.6^\circ\right)$$

[18]Gardner, M. F. and Barnes, J. L., Transients in Linear Systems, N. Y., John Wiley and Sons, 1947, Transform No. 1.304, p. 342.

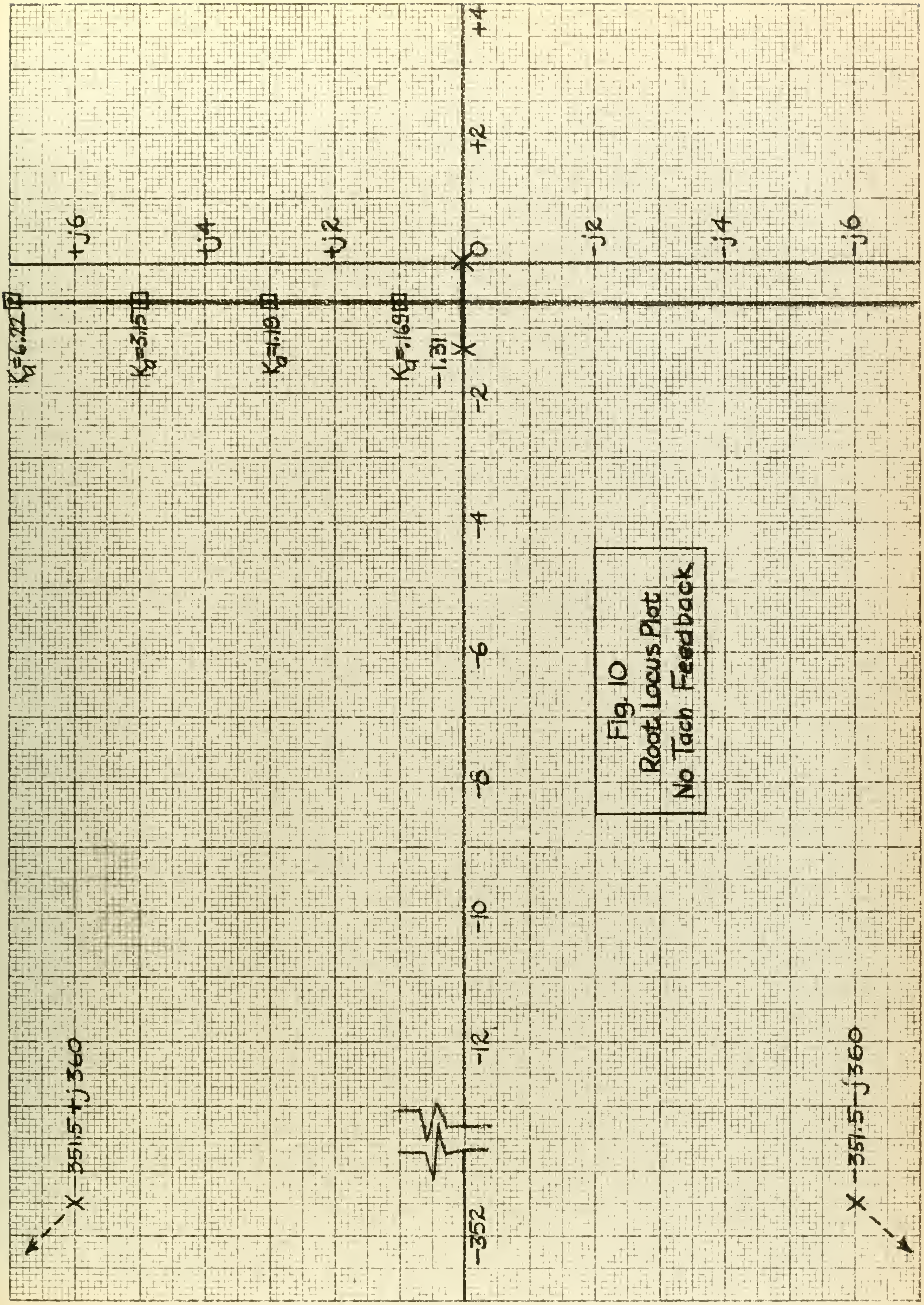


Fig. 10
Root Locus Plot
No Tach Feedback

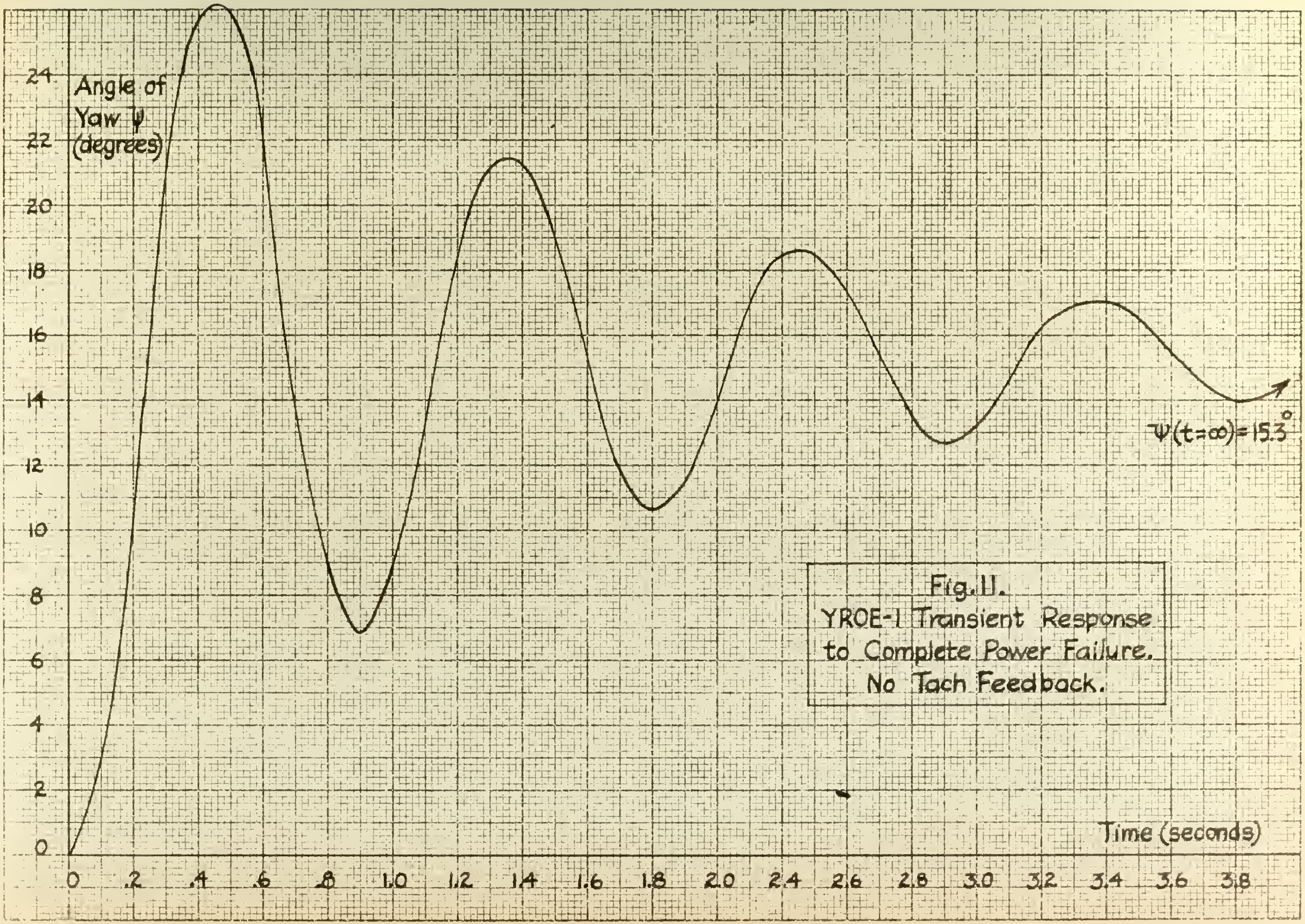


Fig. 11.
YROE-1 Transient Response to Complete Power Failure. No Tach Feedback.

A plot of the transient response defined by this equation is shown in Fig. 11 and it is seen that the response is poorly damped and completely unsatisfactory. The difficulty is obviously caused by the vertical root locus segment being too close to the imaginary axis. Apparently some form of compensation should be utilized which will move this segment away from the imaginary axis and make possible an improved damping ratio with a value of K_a which produces satisfactory steady-state response.

If the open-loop pole at -1.31 is cancelled and replaced with a pole further to the left on the negative real axis, the root locus segment will be moved to the left a corresponding distance. This can be accomplished by a phase lead filter with a zero at -1.31 and a pole at -13.1. The 10 to 1 ratio of pole to zero is a maximum value fixed by well-known practical limitations for a single passive R-C filter.

When such a filter is placed in the main transmission path, the block diagram of the system becomes that shown in Fig. 12.

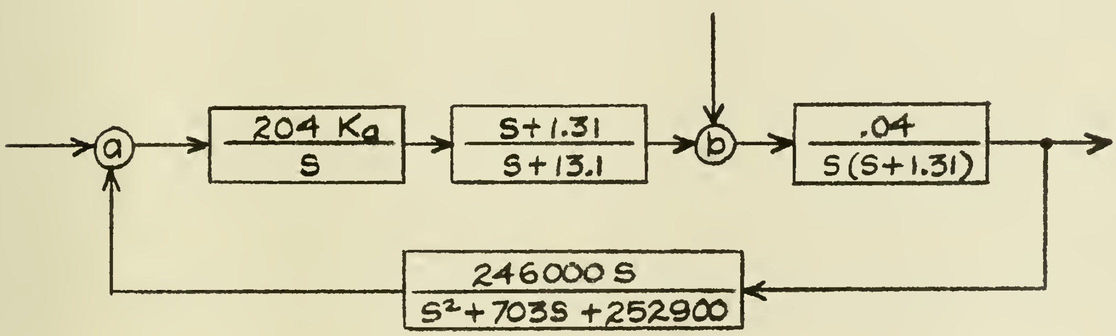


Fig. 12

System Block Diagram with Phase Lead Filter

The open-loop transfer function is now

$$\frac{(20 + K_a)(.04)(246\,000)\,s\,(s+1.31)}{s^2(s+1.31)(s+13.1)(s^2+703s+252\,900)}$$

$$= \frac{2{,}010{,}000\ K_a\,s\,(s+1.31)}{s^2(s+1.31)(s+13.1)(s^2+703s+252900)}$$

and the root locus plot is shown in Fig. 13. The closed-loop transfer function is

$$\frac{\dfrac{.04}{s(s+1.31)}}{1 + \dfrac{2{,}010{,}000\ K_a\,s\,(s+1.31)}{s^2(s+1.31)(s+13.1)(s^2+703s+252900)}}$$

$$= \frac{.04\,(s+13.1)\,(s^2+703s+252\,900)}{s(s+1.31)(s+13.1)(s^2+703s+252900)+2{,}010{,}000\,K_a\,(s+1.31)}$$

and if the Final Value Theorem is applied as before, the steady-state response is given by

$$\frac{16.6}{K_a}.$$

Thus any desired steady-state output requires 10 times the gain required before the lead filter was added. This obviously results from the 10 to 1 ratio of the filter. With K_a equal to 60.3 $\frac{ma}{volt}$ the steady-state output is 15.7 degrees and the complex roots are located at $-6.5 \pm j\,21$. It is noted, however, that a root is now also present at -1.31 because the pole and zero at that location constitute a point segment of root locus. This situation cannot be avoided because, although the filter appears to be in the main path, it is actually in the feedback path with respect to

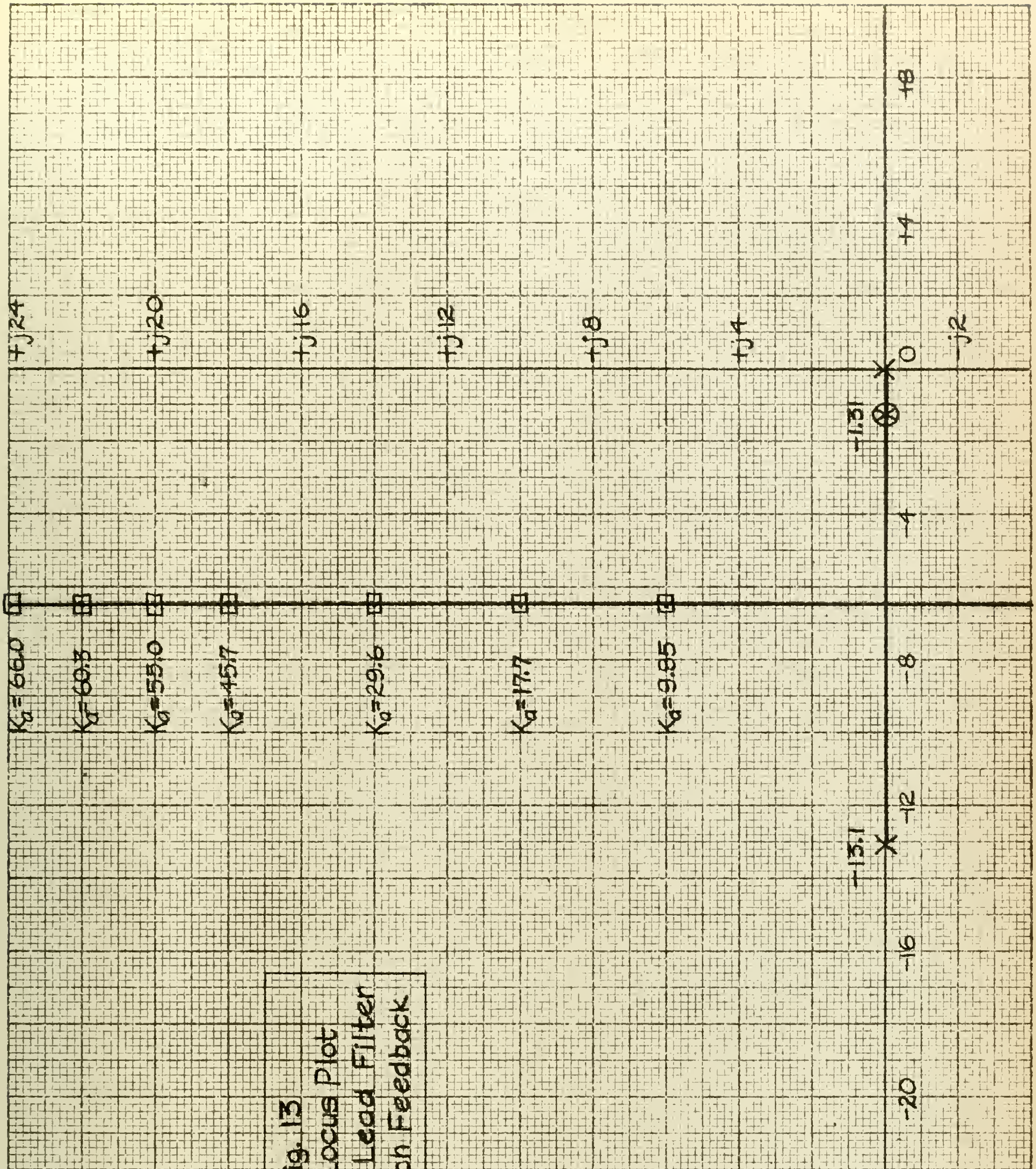


Fig. 13
Root Locus Plot
Phase Lead Filter
No Tach Feedback

an input at node b. Elimination of the pole at -1.31 can only be accomplished by insertion of a zero in the path between node b and the output since this constitutes the entire main path for the input under consideration. No filter can be inserted in this portion of the path, however, because the path consists only of the resultant torque, the airframe, and the output. For the case of a command input, however, the real pole at -1.31 will be cancelled by the zero of the lead filter since, with respect to an input at node a, the filter is in the main transmission path. Hence, for a command input, the two complex conjugate roots will determine the response of the system even though the response to a disturbance input will be determined by the real root at -1.31.

With the system gain set at 60.3 (producing a steady-state output of 15.7 degrees), the closed loop transfer function for a disturbance input is (again neglecting the two pairs of poles and zeros far to the left)

$$\frac{.04\,(s+13.1)}{(s+1.31)(s+6.5+j21)(s+6.5-j21)}$$

and for a 330 ft-lb step torque disturbance input, the transform of the response is

$$\psi(s) = \frac{13.2\,(s+13.1)}{s\,(s+1.31)(s+6.5\pm j21)} = \frac{13.2\,(s+13.1)}{s\,(s+1.31)\left[(s+6.5)^2+(21)^2\right]}$$

Taking the inverse transform,[19] the yaw angle as a function of time is given by

$$\psi(t) = 0.273 + 0.254\,e^{-1.31t} + 0.029\,e^{-6.5t}\sin(21t - 138.6^\circ)$$

This response is shown in Fig. 14 and it is seen that the effect of the sinusoidal term is negligible because of the relatively large negative exponent. The response shown appears to be quite desirable for this type

[19]Ibid., Transform No. 1.320, p. 344.

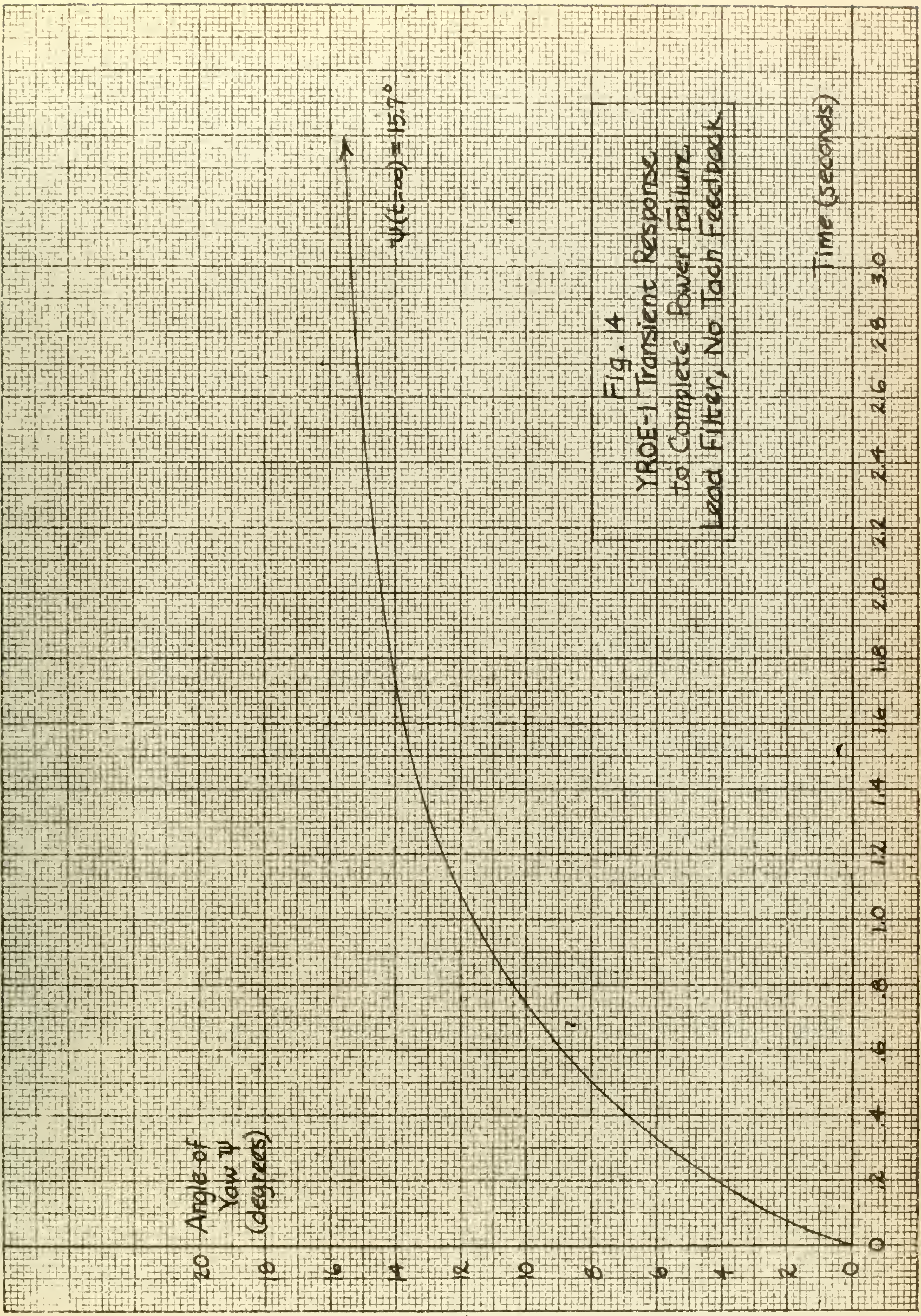


Fig. 4
YROE-1 Transient Response to Complete Power Failure, Lead Filter, No Tach Feedback

of input. The maximum yaw rate is about 0.6 radians per second at the very beginning of the response and decreases thereafter until the steady-state output of 15.7 degrees is reached at about 3 seconds after the disturbance occurs.

A change of torque corresponding to an increase in engine power from zero to one hundred per cent (330 ft-lb) in 0.2 second can be illustrated as shown in Fig. 15.

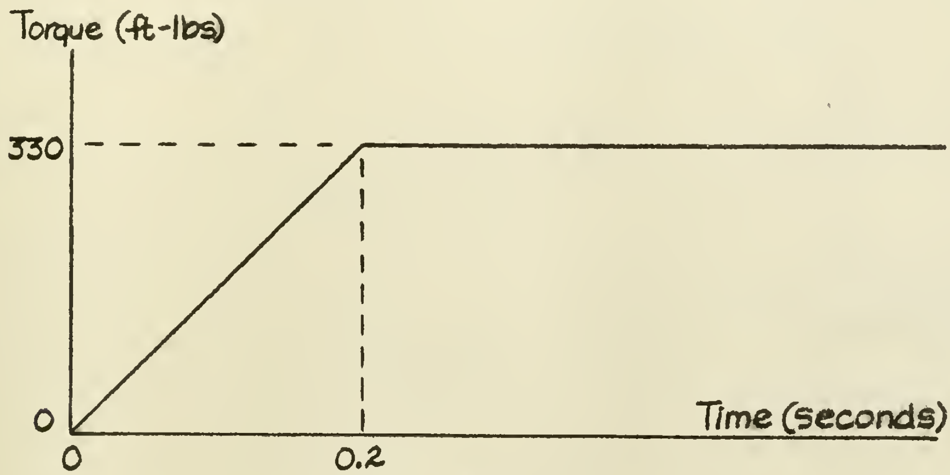


Fig. 15

Torque Change for Full Power Application in 0.2 Second

This power application can be characterized as a ramp torque disturbance input from zero to 0.2 seconds and as the difference of two ramp inputs after 0.2 seconds. Between zero and 0.2 seconds torque is applied at the following rate

$$\frac{330 \text{ ft-lb}}{0.2 \text{ sec}} = 1650 \frac{\text{ft-lb}}{\text{sec}}$$

or, the torque on the airframe at any time between these limits is given by

$$1650t \text{ ft-lb}$$

The Laplace Transform of this expression is

$$\frac{1650}{s^2} .$$

After 0.2 seconds the disturbance input is represented by the sum of a positive ramp of magnitude 1650 ft-lb/sec starting at zero time and a negative ramp of the same magnitude starting at 0.2 seconds. This is illustrated in Fig. 16.

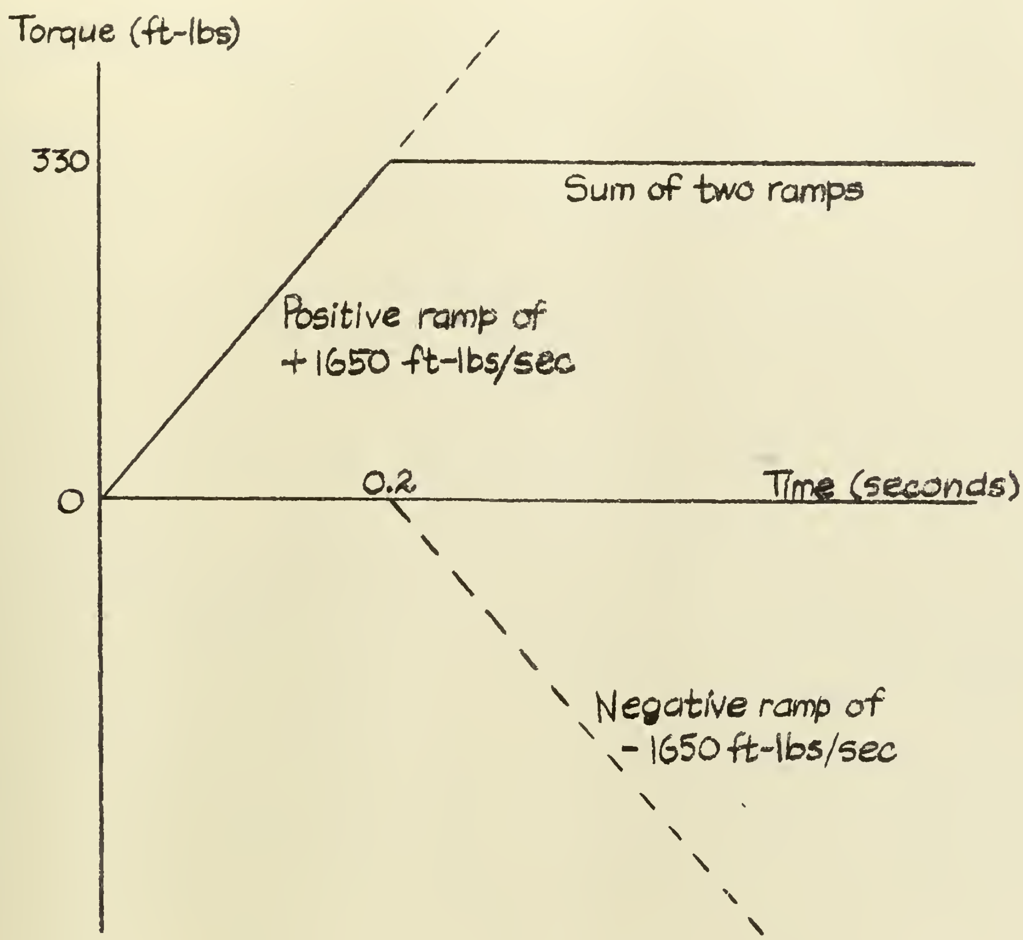


Fig. 16

Sum of Two Ramps to Represent Power Application

It is apparent that the system response at any time after 0.2 seconds can be obtained by computing the response to a single ramp which started at zero time and from this value subtracting the response to another ramp

which started at 0.2 seconds. The response to each individual ramp is identical except for a translation of 0.2 seconds and a change of sign, so it is only necessary to actually compute a response for one ramp. If the value of the output at time 0.6 seconds (for example) is desired, it is only necessary to subtract the response to the single ramp at time 0.4 seconds from the response to the single ramp at time 0.6 seconds.

For a single ramp torque disturbance input to the system presently being considered, the transform of the output is

$$\psi(s) = \frac{.04(s+13.1)}{(s+1.31)(s+6.5\pm j21)}\left(\frac{1650}{s^2}\right)$$

$$= \frac{66(s+13.1)}{s^2(s+1.31)(s+6.5\pm j21)}$$

and expansion by the partial fraction method[20] yields

$$\psi(s) = -\frac{0.974}{s} + \frac{1.362}{s^2} + \frac{0.966}{s+1.31} + \frac{0.00614(s-1.8)}{(s+6.5)^2+(21)^2}$$

and the inverse transform is

$$\psi(t) = -0.974 + 1.362\,t + 0.966\,e^{-1.31t} + 0.0066\,e^{-6.5t}\sin(21t+112^\circ)$$

The transient response obtained from this expression by the method previously explained is shown in Fig. 17.

For the torque condition corresponding to power application from zero to one hundred per cent in 2.0 seconds the previously determined transient response equation can be used if each coefficient is divided by 10. This simple procedure is possible because the magnitude of the ramp in this case is

$$\frac{330 \text{ ft-lb}}{2.0 \text{ sec}} = 165 \frac{\text{ft-lb}}{\text{sec}}$$

which is one-tenth of the magnitude of the previous ramp. Thus the tran-

[20] Ibid., p. 154-163

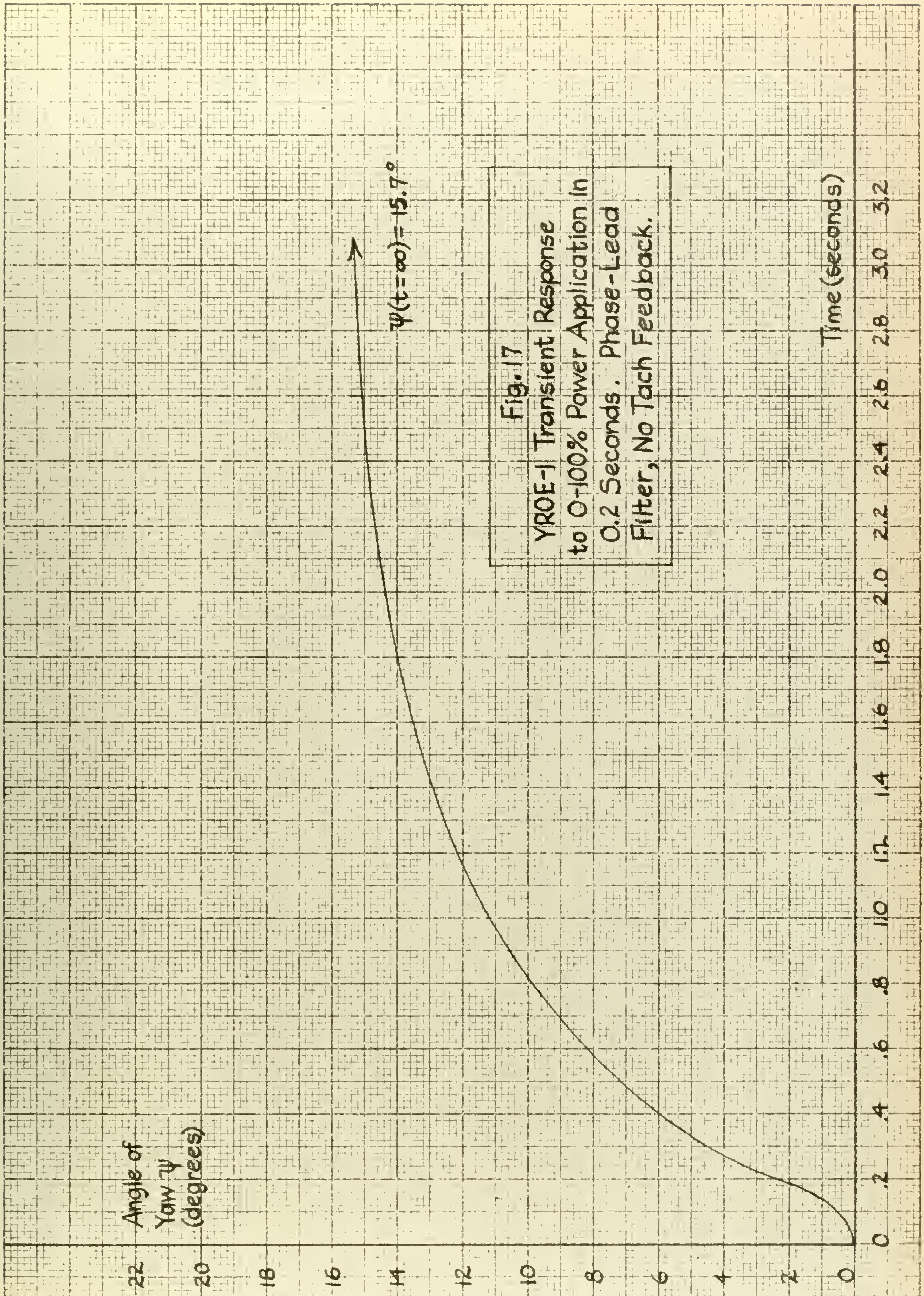
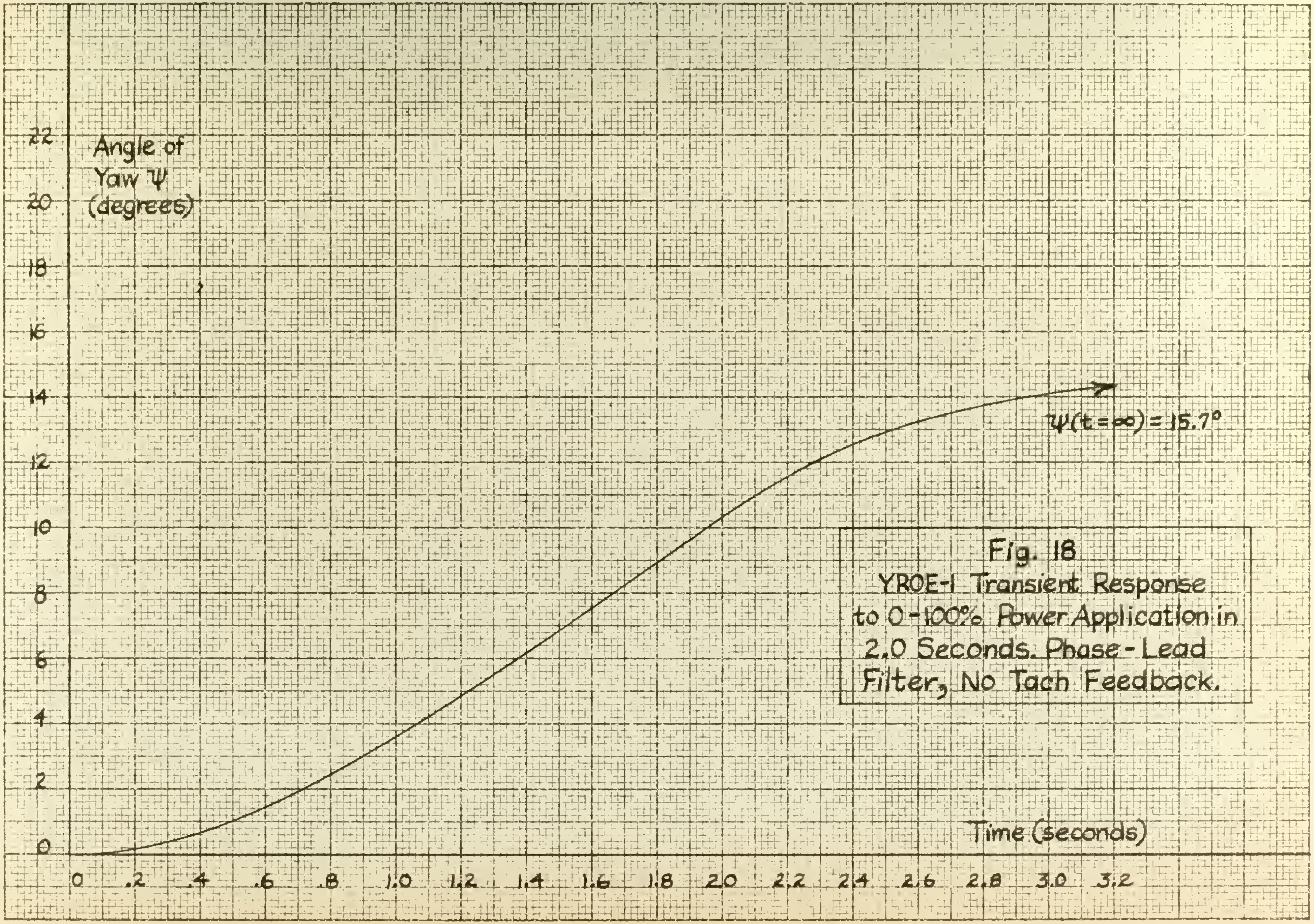


Fig. 17
YROE-1 Transient Response to 0-100% Power Application In 0.2 Seconds. Phase-Lead Filter, No Tach Feedback.
ψ(t=∞)=15.7°
Angle of Yaw ψ (degrees)
Time (seconds)
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
0
2
4
6
8
10
12
14
16
18
20
22


Angle of
Yaw ψ
(degrees)
22
20
18
16
14
12
10
8
6
4
2
0
ψ(t=∞)=15.7°
Fig. 18
YROE-1 Transient Response
to 0-100% Power Application in
2.0 Seconds. Phase-Lead
Filter, No Tach Feedback.
Time (seconds)
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2

sient response is obtained from

$$\psi(t) = -0.0974 + 0.1362\,t + 0.0966\,e^{-1.31t} + 0.00066\,e^{-6.5t}\sin(21t + 112^\circ)$$

and is shown in Fig. 18.

It is interesting to note, in comparing the transient response for the three conditions of disturbance torque application, that the steady-state value is 15.7 degrees for all three cases.

In considering a command input to this system, node b may be eliminated, and with the same value of K_a equal to 60.3, the system block diagram of Fig. 12 reduces to that shown in Fig. 19.

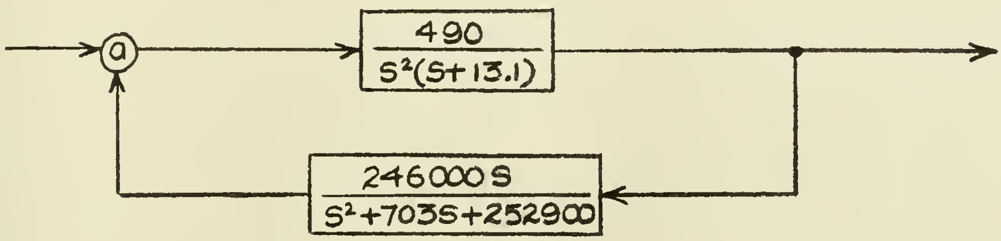


Fig. 19
System Block Diagram for Command Input
(Phase-Lead Filter, No Tachometer)

The closed-loop transfer function for this condition is

$$\frac{\dfrac{490}{s^2(s+13.1)}}{1 + \left[\dfrac{490}{s^2(s+13.1)}\right]\left[\dfrac{246{,}000\, s}{s^2+703s+252900}\right]}$$

$$= \frac{490\,(s^2+703s+252\,900)}{s^2(s+13.1)(s^2+703s+252\,900) + 490(252900)}$$

$$\cong \frac{490}{s(s+6.5+j21)(s+6.5-j21)} = \frac{490}{s\left[(s+6.5)^2+(21)^2\right]}$$

It now becomes necessary to choose an appropriate forcing function to represent a command input in order that a transient response may be calculated and the possible presence of undesirable oscillations detected. If the command input is chosen as a step of 1 volt magnitude, the transform of the response is

$$\psi(s) = \frac{490}{s^2\left[(s+6.5)^2 + (21)^2\right]}$$

and the inverse transform is[21]

$$\psi(t) = -0.0274 + 1.02t + 0.0485\, e^{-6.5t} \sin(21t - 214^\circ)$$

The response represented by this expression is shown in Fig. 20. It is seen that the sinusoidal term has negligible effect and that, for all practical purposes, a constant turning rate of about one radian per second is produced. This particular rate is not significant because an adjustment of the rudder pedal synchro gain will change the magnitude of the voltage produced by a given pedal deflection, and hence will change the yaw rate produced by the pedal deflection. It is significant, however, that a steady yaw rate without oscillation results from a command step input.

This does not necessarily mean that oscillations may not occur when the command input is of the type actually inserted by the pilot when he desires to change from one specific heading to another. Characterizing this type of input by a mathematical expression is extremely difficult, if not impossible. Some basis is needed, however, on which to compare the responses of different systems to this general type of input. Accordingly, an impulse input was selected as this basis, even though

[21] Ibid., Transform No. 2.601, p. 350.

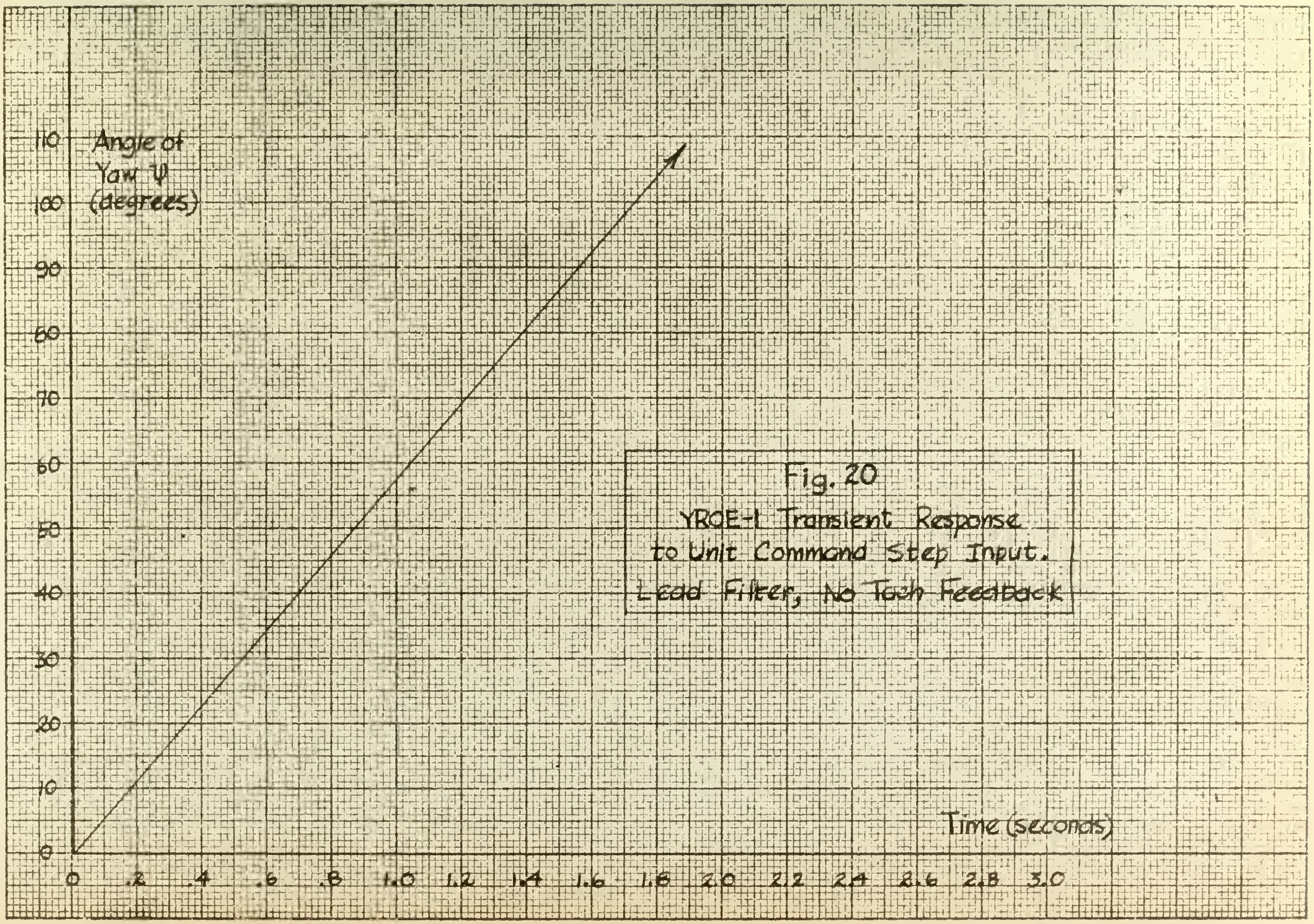


Fig. 20
YRCE-1 Transient Response
to Unit Command Step Input.
Lead Filter, No Tach Feedback

it is fully realized that an impulse cannot be justified as representing an actual command input.

When subjected to a command impulse forcing function, the transform of the response is

$$\psi(s) = \frac{490}{s\left[(s+6.5)^2+(21)^2\right]}$$

and the inverse transform is[22]

$$\psi(t) = 1.014 + 1.06\, e^{-6.5t} \sin(21t - 107^\circ)$$

The transient response given by this expression is shown in Fig. 21. Again it should be emphasized that no particular significance can be attached to this transient response except in comparison with the response of another system to the same input. One exception to this statement is the obvious fact that the response demonstrates that the system does achieve a finite steady-state value, that is, the system is stable.

The system just analyzed appears to be generally satisfactory although it does not meet the Hiller Aircraft specification of a maximum deviation from course of 5 degrees in response to a change in torque from zero to 330 ft-lb in 2.0 seconds. This value of steady-state output could be obtained by increasing the system gain but the gain is already relatively high at 60.3 ma/volt. It was therefore decided to investigate the possibility of further improving system performance and lowering the required gain through insertion of a phase lag filter in series with the previously added phase lead filter. The effects of several different phase lag-pole-zero configurations on the root-locus were considered. A lag filter with a pole at -0.2 and a zero at -3.0 appeared to warrant further consideration. A block diagram of the sys-

[22]Ibid., Transform No. 1.304, p. 342.

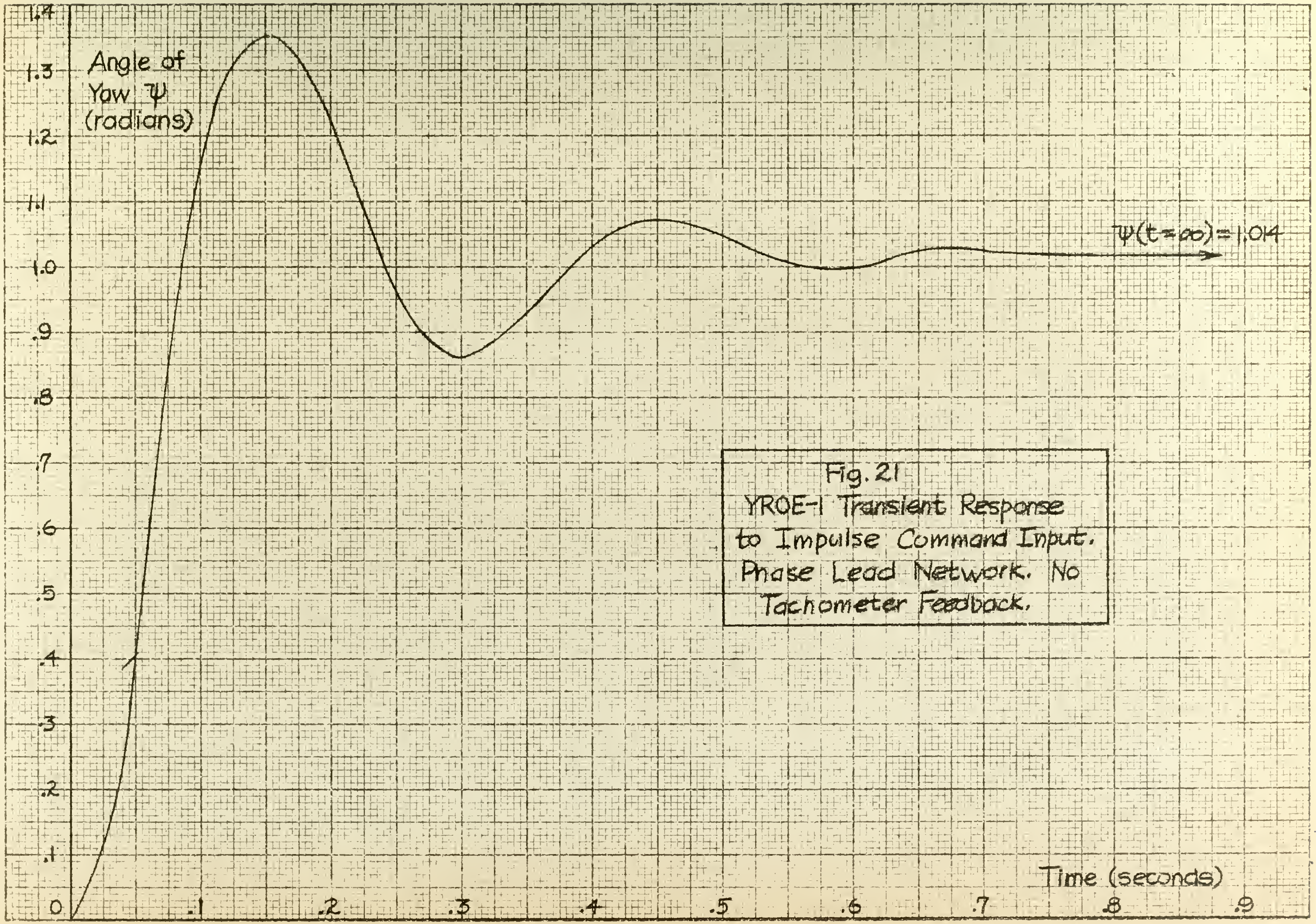


Fig. 21
YROE-1 Transient Response to Impulse Command Input. Phase Lead Network. No Tachometer Feedback.

tem with this filter added is shown in Fig. 22.

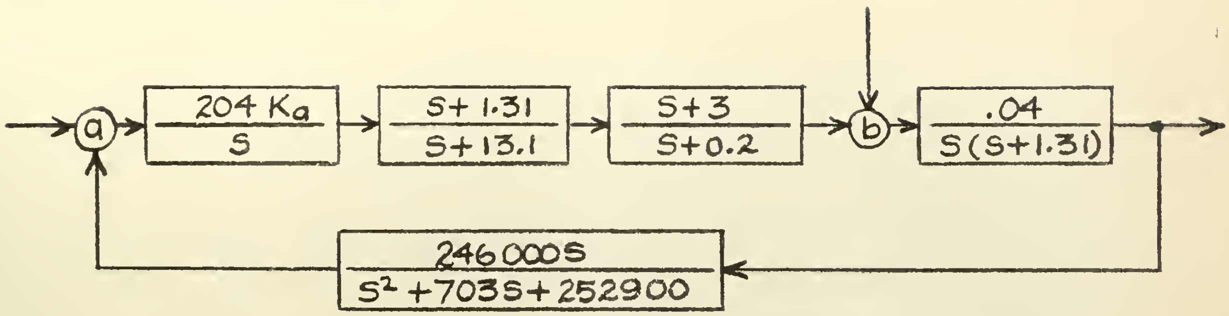


Fig. 22

System Block Diagram with Lead-Lag Filters

The open-loop transfer function for this system is

$$\frac{(204\,K_a)(s+1.31)(s+3)(.04)(246{,}000\,s)}{s^2(s+0.2)(s+1.31)(s+13.1)(s^2+703s+252900)}$$

$$= \frac{2{,}010{,}000\,K_a\,s(s+1.31)(s+3)}{s^2(s+0.2)(s+1.31)(s+13.1)(s^2+703s+252900)}$$

and the root locus plot is shown in Fig. 23. A comparison of this plot with the root locus shown in Fig. 13 for the system with phase lead filter only shows the possibility of placing the complex conjugate roots in a more desirable location. The real root at -1.31 will still be present, however, for any input at node b, for the reasons previously set forth.

If the complex conjugate roots are placed at $-4.5\pm j6.5$ the required value of K_a is 12.1 ma/volt. The additional real root introduced by the phase lag filter occurs at -5.0. The closed loop transfer function is

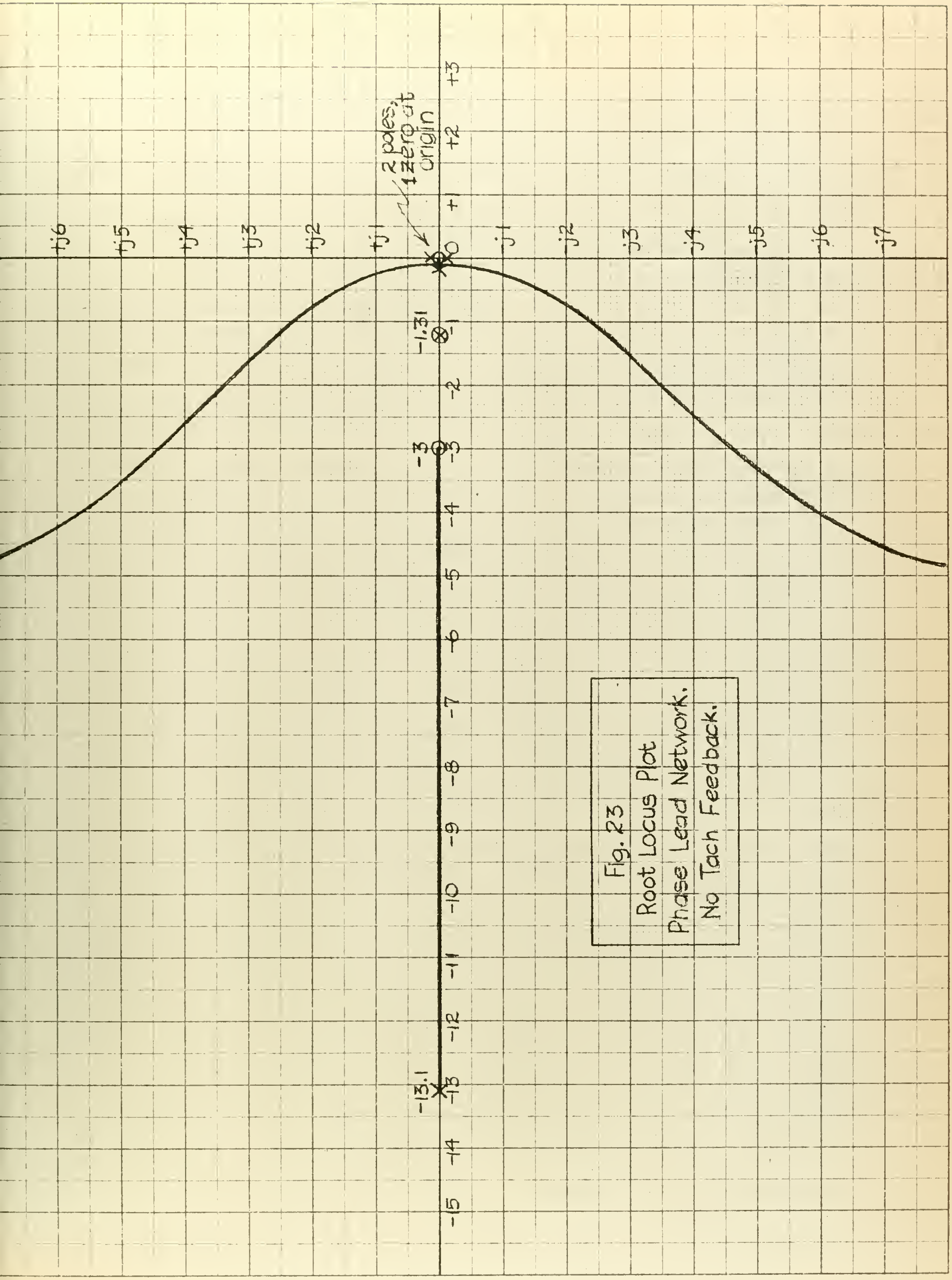


Fig. 23
Root Locus Plot
Phase Lead Network.
No Tach Feedback.
2 poles; 1 zero at origin
-1.31
-3
-13.1
+j6
+j5
+j4
+j3
+j2
+j1
0
-j1
-j2
-j3
-j4
-j5
-j6
-j7
+3
+2
+1
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
-13
-14
-15

now

$$\frac{.04(s+0.2)(s+13.1)}{(s+1.31)(s+5)(s+4.5+j6.5)(s+4.5-j6.5)}$$

and the transform of the response to a 330 ft-lb step torque disturbance is

$$\psi(s) = \frac{13.2(s+0.2)(s+13.1)}{s(s+1.31)(s+5)(s+4.5+j6.5)(s+4.5-j6.5)}$$

Expansion by partial fractions yields

$$\psi(s) = \frac{0.0844}{s} + \frac{0.681}{s+1.31} - \frac{0.654}{s+5} - \frac{0.096(s+35)}{(s+4.5)^2+(6.5)^2}$$

and the inverse of this expression is

$$\psi(t) = 0.0844 + 0.681e^{-1.31t} - 0.654e^{-5t} - 0.46e^{-4.5t}\sin(6.5t+12^\circ)$$

The transient response represented by this equation is shown in Fig. 24. The steady-state yaw angle is only 4.8 degrees but an overshoot of 23 degrees occurs at 0.5 second. The maximum yaw rate is slightly over one radian per second and this does not satisfy Hiller Aircraft specifications of 0.6 radians per second but Hiller engineers have stated that this rate could be tolerated because of its short duration. The large overshoot does not result from the sinusoidal term which becomes negligible after 0.3 second but results from the combination of the two exponential terms.

Following the same procedures as previously described for the cases of power application in 0.2 second and 2.0 seconds, the transient responses of Fig. 25 and Fig. 26 were obtained. Again it is noted that the steady-state output is the same (4.8 degrees) in all three cases. Power application in 0.2 second produces a transient response almost identical to that of power failure. Power application in 2.0 seconds causes a peak overshoot of 13.5 degrees at 2.2 seconds and the maximum rate is approximately equal to the specification of 0.2 radians per second.

It is interesting at this point to investigate the response to a

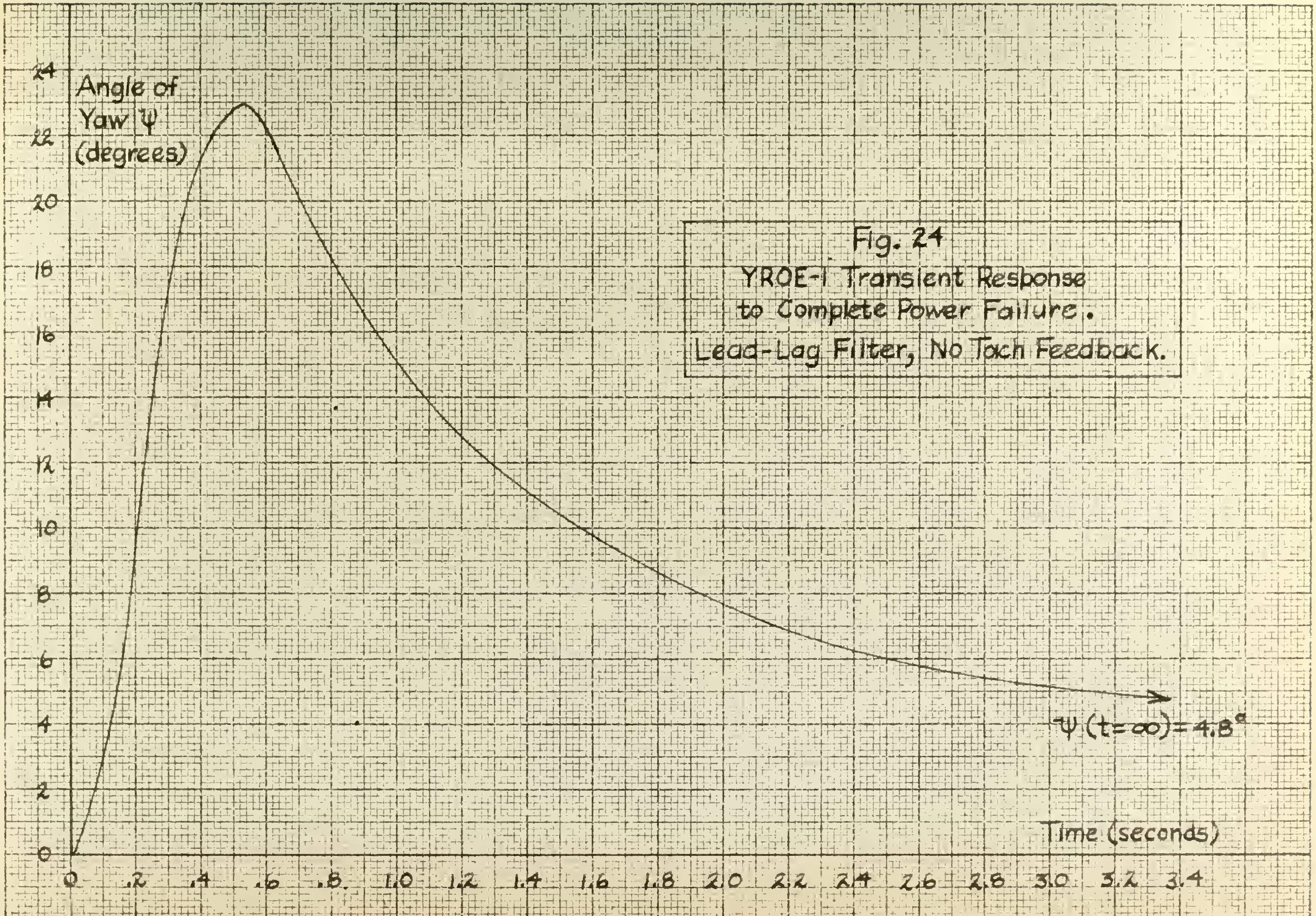


Fig. 24
YROE-1 Transient Response to Complete Power Failure.
Lead-Lag Filter, No Tach Feedback.

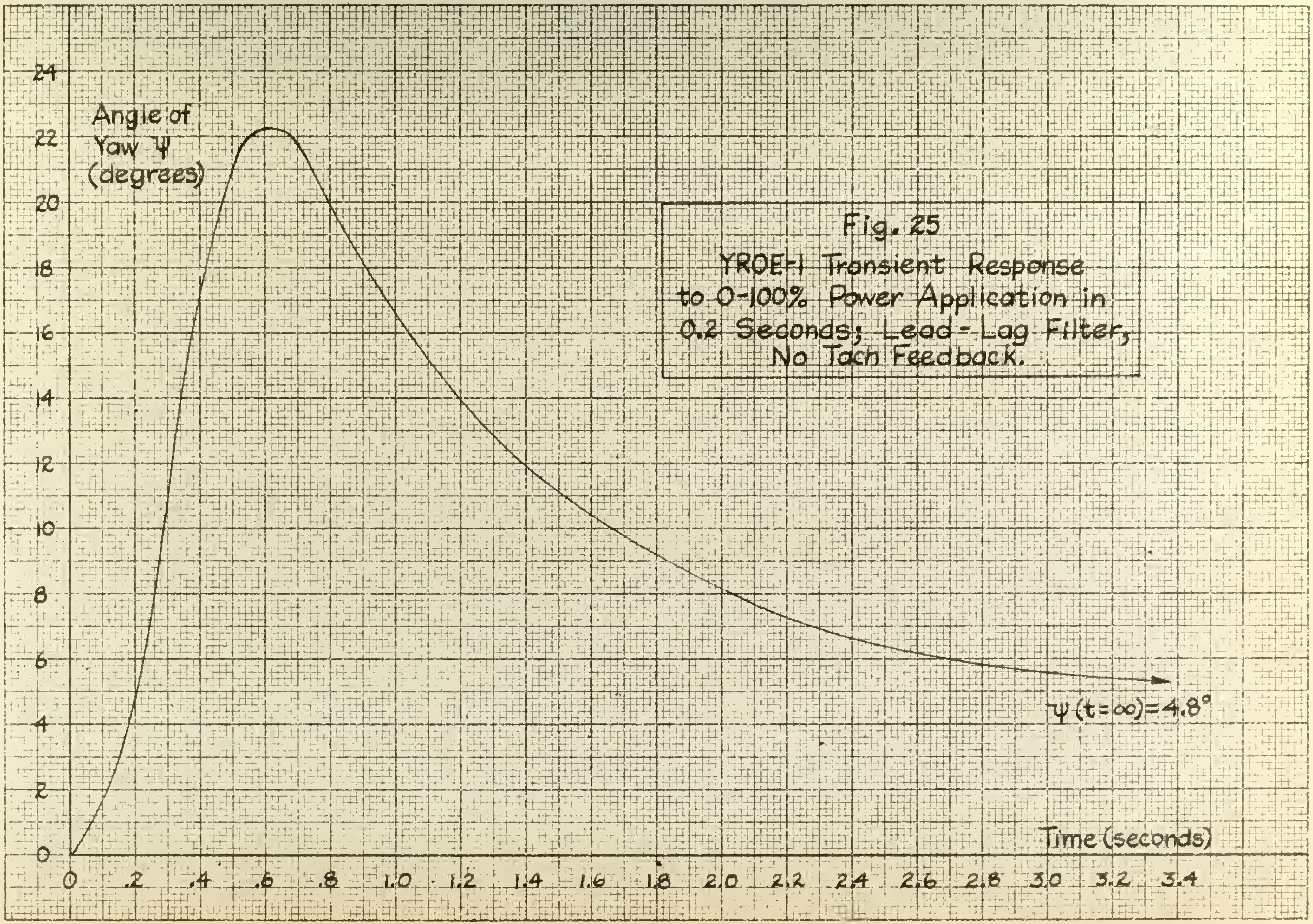


Fig. 25
YROE-1 Transient Response to 0-100% Power Application in 0.2 Seconds; Lead-Lag Filter, No Tach Feedback.

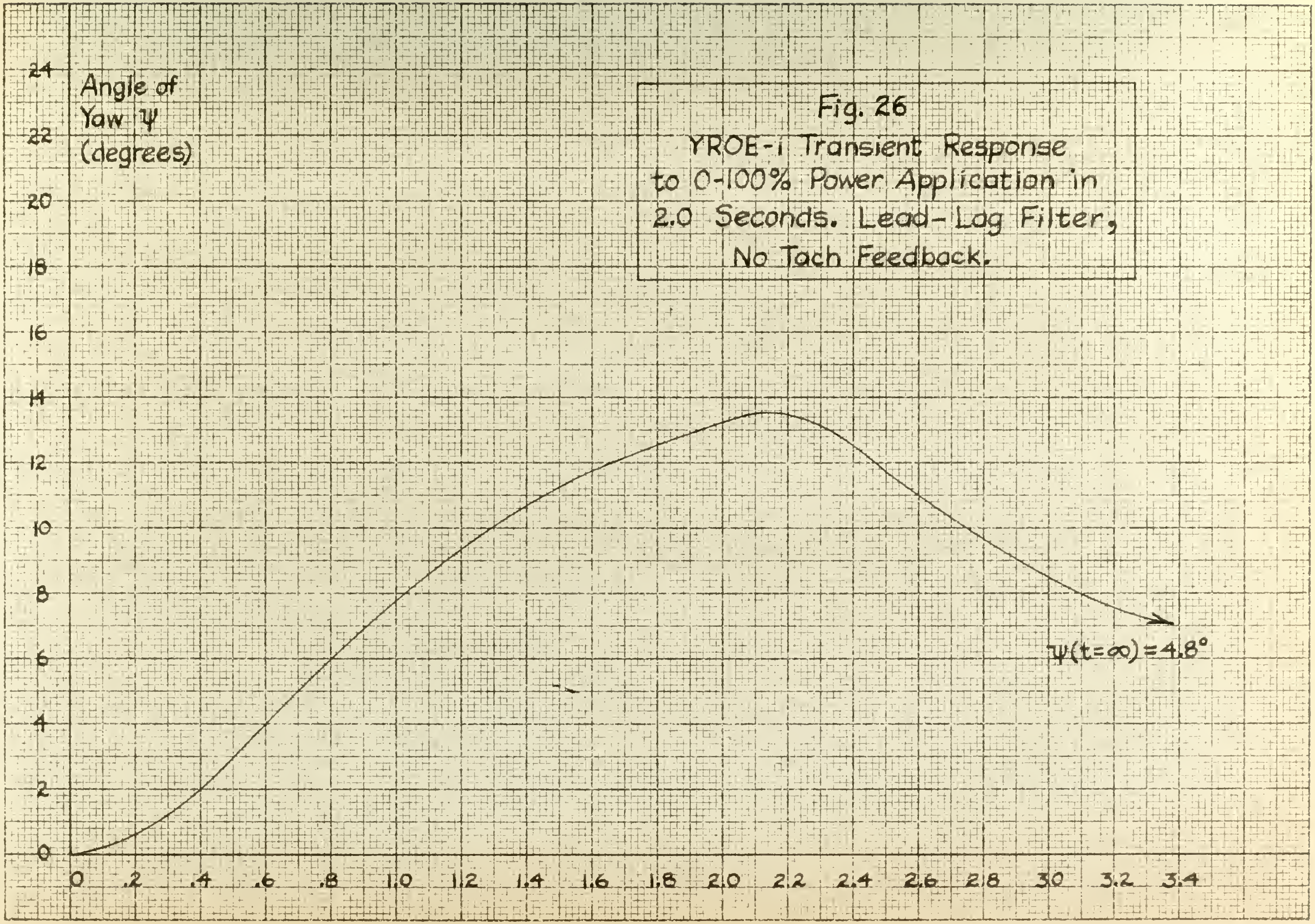


Fig. 26
YROE-1 Transient Response to 0-100% Power Application in 2.0 Seconds. Lead-Lag Filter, No Tach Feedback.

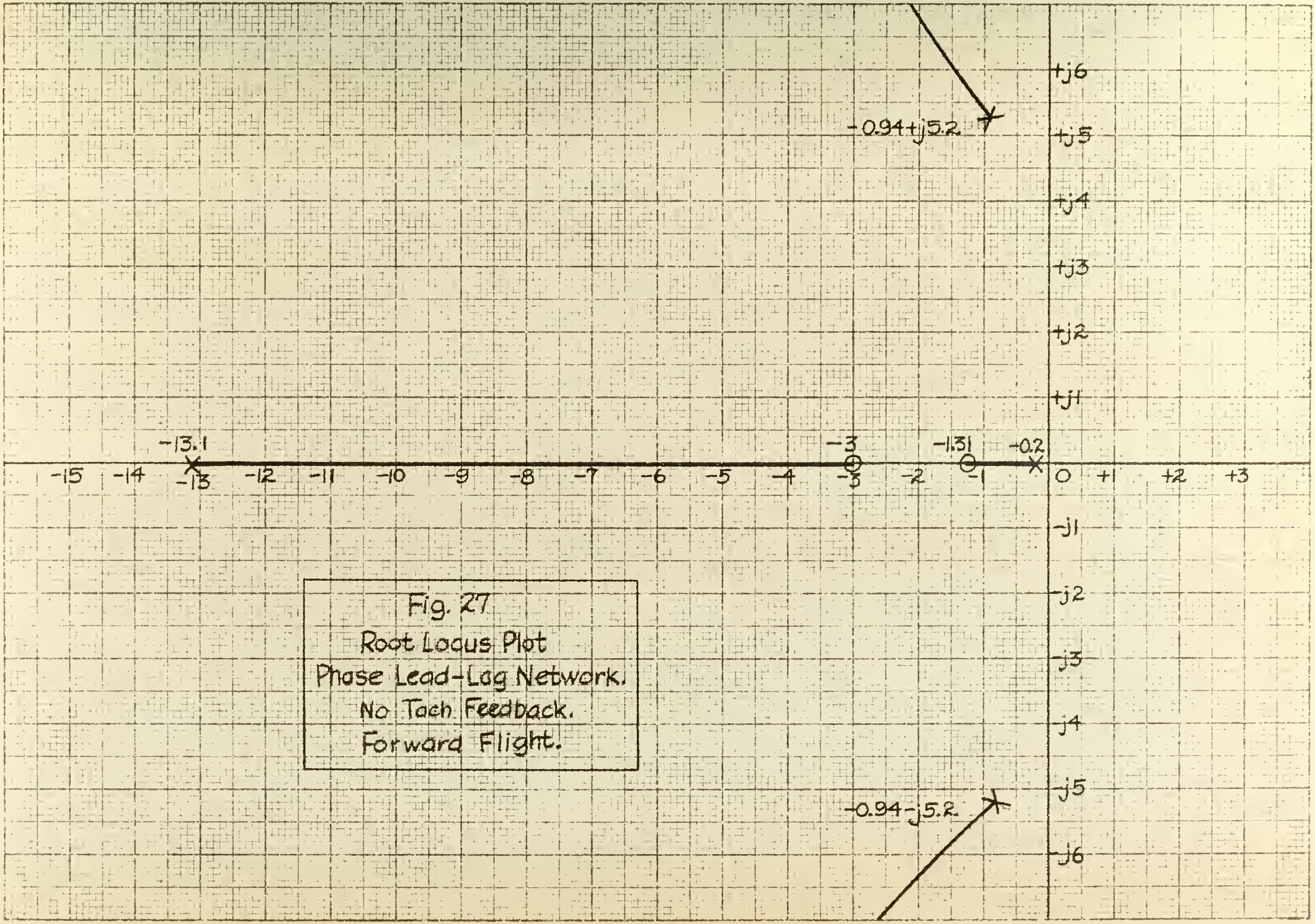


Fig. 27
Root Locus Plot
Phase Lead-Lag Network.
No Tach Feedback.
Forward Flight.

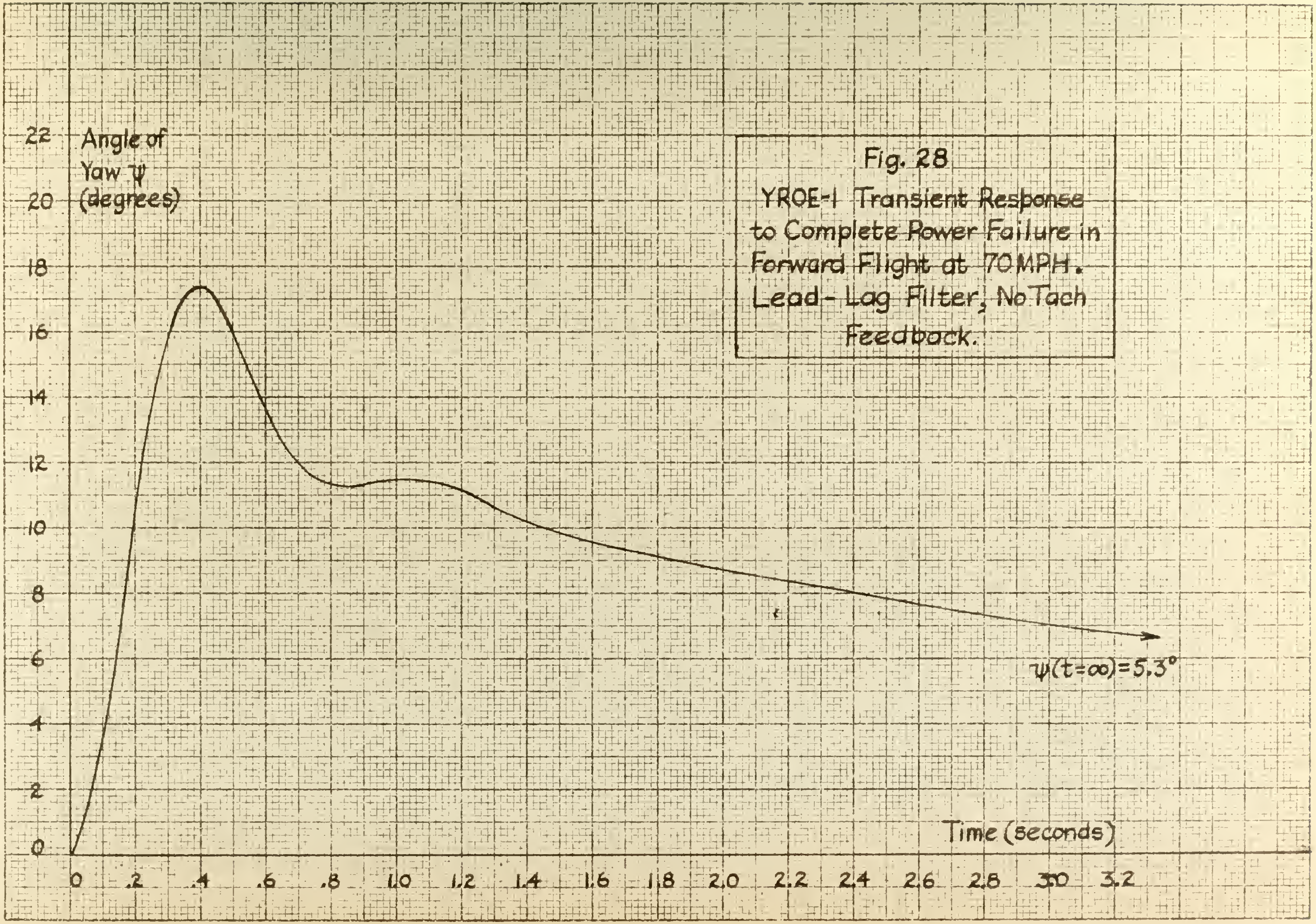


Fig. 28
YROE-1 Transient Response to Complete Power Failure in Forward Flight at 70 MPH. Lead-Lag Filter, No Tach Feedback.

power failure while the helicopter is in forward flight at a velocity of 70 miles per hour. It was stated earlier that the yaw experienced due to a torque disturbance while in forward flight must be less than that experienced for the same disturbance while in hovering flight because of the presence of a weathervaning effect while in the former flight condition. As set forth in Section 4, the transfer function for the helicopter airframe in this condition is

$$\frac{.04}{s^2 + 1.87s + 27.5} = \frac{.04}{(s+0.94+j5.2)(s+0.94-j5.2)}$$

The open-loop transfer function is now

$$\frac{2{,}010{,}000\, K_2 (s+1.31)(s+3)}{(s+0.2)(s+13.1)(s+0.94 \pm j5.2)(s^2+703s+252900)}$$

and the root locus plot is shown in Fig. 27. With the same value of gain (12.1) the complex conjugate roots are now located at $-3.2 \pm j8.5$, and the two real roots are at -0.6 and -7.6. The transient response is given by

$$\psi(t) = 0.092 + 0.20e^{-0.6t} - 0.11e^{-7.6t} - 0.24e^{-3.2t}\sin(8.5t + 51^\circ)$$

and is shown plotted in Fig. 28. The steady-state output has increased slightly to 5.3 degrees but the peak overshoot has decreased from 23 degrees to 17.5 degrees which bears out the previous assumption.

Returning now to consideration of the hovering condition, the system block diagram may be reduced to that shown in Fig. 29 for the case of a command input.

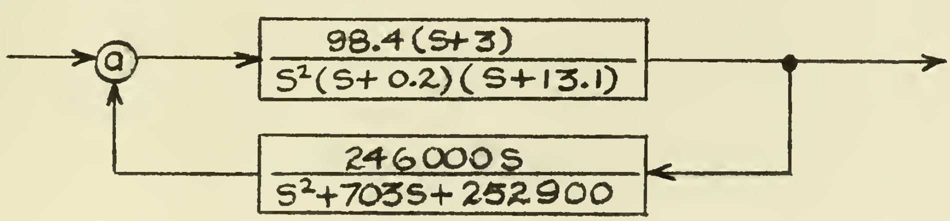


Fig. 29

System Block Diagram for Command Input

(Lead-Lag Filter, No Tachometer)

the closed-loop transfer function for this condition is

$$\frac{98.4(s+3)}{s(s+5)(s+4.5\pm j6.5)} = \frac{98.4(s+3)}{s(s+5)\left[(s+4.5)^2+(6.5)^2\right]}$$

and the transform of the response to a one volt command step input is

$$\dot{\psi}(s) = \frac{98.4(s+3)}{s^2(s+5)\left[(s+4.5)^2+(6.5)^2\right]}$$

$$= -\frac{0.0072}{s} + \frac{0.945}{s^2} - \frac{0.1853}{s+5} + \frac{0.1916(s+9.56)}{(s+4.5)^2+(6.5)^2}$$

The inverse transform is

$$\dot{\psi}(t) = -0.007 + 0.945t - 0.185e^{-5t} + 0.241e^{-4.5t}\sin(6.5t+52°)$$

and the transient response is shown in Fig. 30. It is seen that this system produces practically the same turning rate (about one radian per second) as the system with lead filter only. An insignificant sinusoidal oscillation is present in the first 0.4 second. The same comments regarding adjustment of turning rate that were made for the previous system apply here.

Representing the command input as an impulse, the transform of the output is

$$\dot{\psi}(s) = \frac{98.4(s+3)}{s(s+5)\left[(s+4.5)^2+(6.5)^2\right]}$$

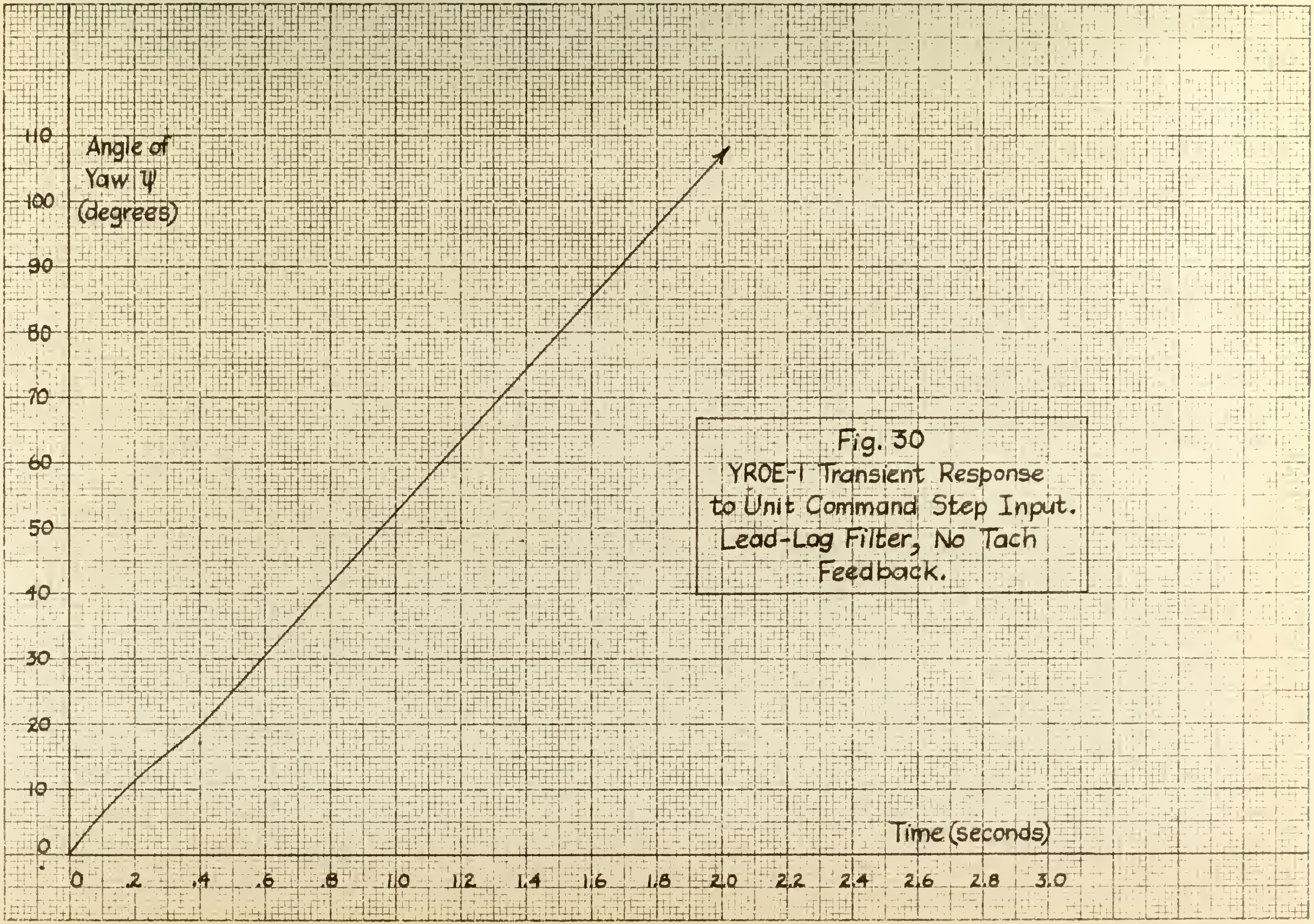


Fig. 30
YROE-I Transient Response to Unit Command Step Input. Lead-Log Filter, No Tach Feedback.

The inverse transform of the expression is[23]

$$\psi(t) = 0.946 + 0.929\,e^{-5t} + 1.96\,e^{-4.5t}\sin(6.5t - 108^\circ)$$

which is shown plotted in Fig. 31. A comparison of this response with that shown in Fig. 21 for the system with lead filter only shows that peak overshoot for the two systems is practically the same but that the lead-only system responds more rapidly and will be more oscillatory than the lead-lag system in response to the same input. It should again be noted that any evaluation of these two plots, except in comparison with each other, will not be indicative of actual response because of the unrealistic nature of the impulse as a command input.

[23]Ibid, Transform No. 1.320, p. 344.

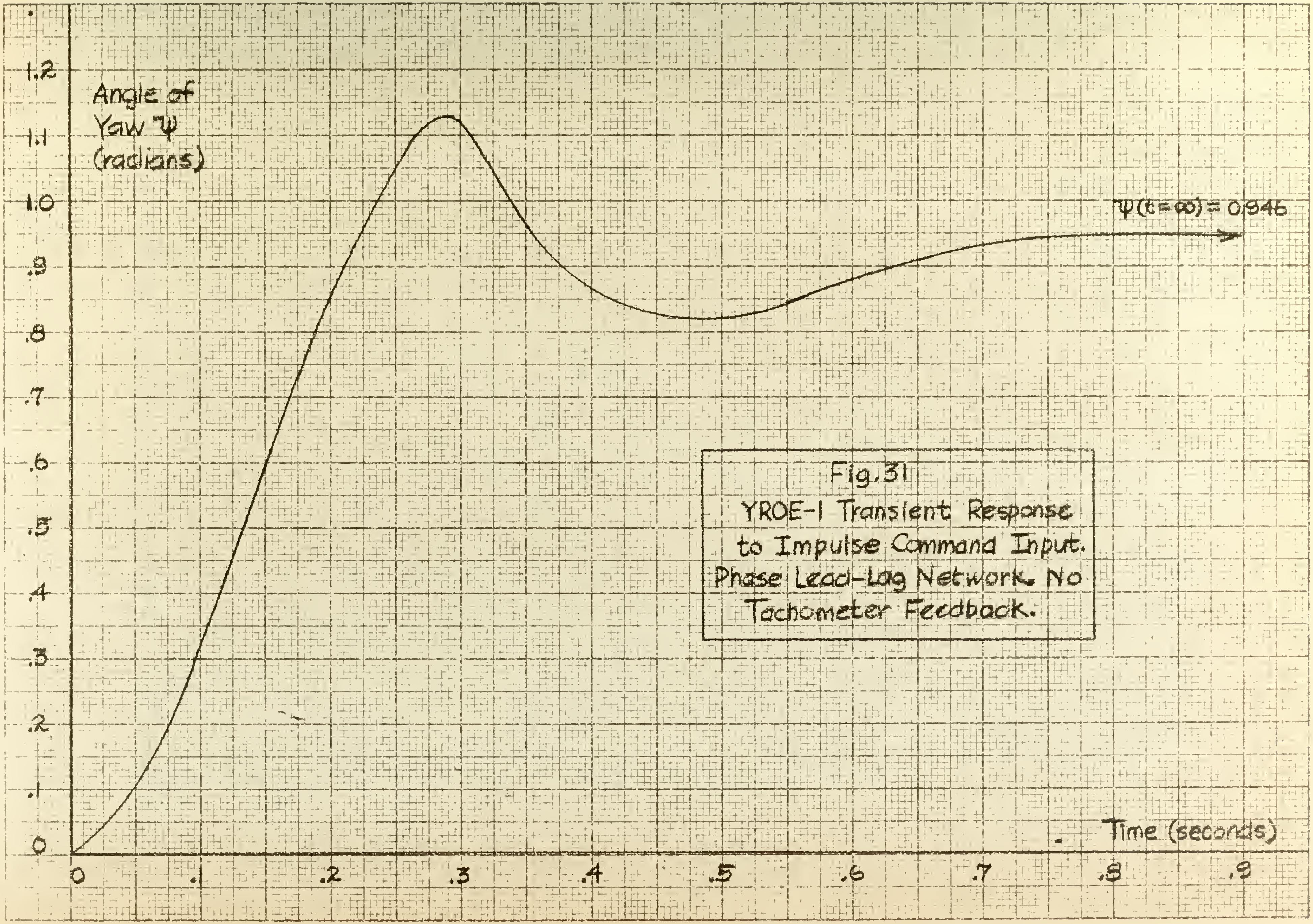


Fig. 31
YROE-1 Transient Response to Impulse Command Input. Phase Lead-Lag Network. No Tachometer Feedback.

6. Discussion of Results and Conclusions.

The yaw stabilization servo system as originally proposed by Royal Industries Incorporated will not produce a satisfactory response without some form of compensation. This results from the dominant complex conjugate roots (closed-loop poles) having a small real part and a large imaginary part. These characteristics produce a yaw output with a high frequency of oscillation which is very slowly damped and completely unsatisfactory. As may be seen from the root locus of Fig. 10 the real part of these dominant complex roots can never exceed -0.65 (without system compensation) because this segment of root locus is a vertical straight line. The imaginary part of the complex roots may be made as small as desired by decreasing system gain but a minimum value of gain is set by steady-state output requirements and this minimum gain locates the roots with a relatively large imaginary part.

The tachometer feedback loop can be eliminated from the system proposed by Royal Industries because it does not improve system performance. The best steady-state output with the least system gain is attained when there is no tachometer feedback. Transient response for a given value of steady-state yaw angle is unaffected by tachometer feedback.

Phase lead compensation will move the vertical segment of root locus to the left as shown in Fig. 13 and increase (in a negative sense) the real part of the complex roots. This compensation will also introduce a real root at -1.31 for any torque disturbance input and this root will be dominant in determining the transient response to this type of input. Relatively high gains are required with this system in order to produce low values of steady-state yaw angle but the transient response character-

istics are excellent for torque disturbance inputs; that is, no overshoot is present and yaw rates are not excessive. For command inputs to this system, the open loop pole at -1.31 is cancelled and the complex roots become dominant in determining transient response. Although the sinusoidal oscillation is rapidly damped, it is of high frequency while present and could conceivably be troublesome.

Addition of a phase lag filter offers possibilities of reshaping the root locus to a more desirable configuration. A lag filter with pole at -0.2 and zero -3.0 makes possible a better placement of the complex roots and a much lower required gain but introduces an additional real root. If the gain is selected so as to produce a steady-state output of about 5 degrees this additional real root is located at -5.0. For a disturbance input the complex roots are again not dominant because of the real root at -1.31. The combination of this root with the one at -5.0 produces a rather severe overshoot early in the response and accompanying high yaw rates. For a command input, the open-loop pole at -1.31 is cancelled as before and the complex roots become dominant. Since these roots have a much smaller imaginary part than in the case of the lead filter only, the sinusoidal oscillation has a lower frequency with comparable damping characteristics and the response to a command input is improved over that with the lead filter only. Steady-state yaw angle resulting from a disturbance input is reduced and lower gain is required. These improvements are obtained, however, at the expense of producing an undesirable overshoot with high yaw rates for disturbance inputs.

Both the lead and lead-lag systems represent acceptable solutions to the YROE-1 yaw stabilization problem. The lead system is considered to

be much the better of the two since the helicopter response to disturbance inputs is greatly decreased. The final yaw angle of 15 degrees for the power increase over 2.0 seconds does not meet the Hiller specifications of 5 degrees but this is considered to be acceptable since this disturbance is the least severe of the three investigated. All other specifications are met or bettered.

This type of analysis can only show general trends of system performance with variations of compensation and system parameters. The amount of numerical computation required in any attempt to optimize such all-important items as filter pole-zero locations and system gain dictates the use of analog computer techniques for analyses more comprehensive than those presented in this paper.

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